

Fig. 2. Unevenness of the road and derived excitation functions

excitations then result in a torsion excitation of the wheel and thus the whole drive too.

Then we use the stiffness of the tyre :

- in vertical direction

$$c_y = c_R \cdot \cos^2 \alpha + c_T \cdot \sin^2 \alpha \quad (1a)$$

- in horizontal direction

$$c_x = c_R \cdot \sin^2 \alpha + c_T \cdot \cos^2 \alpha \quad (1b)$$

where angle α is according to Fig. 1. The wheel (an undeformed tyre) is in the contact with the unevenness at the point C. The location of the point C varies according to the shape of the unevenness and according to the fact whether the wheel is running onto the unevenness or down of it. The order to express that fact we introduce variables or courses by Misun (ref. 1) : $y_E(x)$ - equidistant to the unevenness $y(x)$, $R_x^E(x)$, $R_y^E(x)$, $\alpha(x)$, than we calculate

$$y_e = y_E - R \quad (2)$$

In Fig. 2 these quantities are shown for the unevenness of the double-sinusoid shape.

2. VEHICLE MODEL

2.1. Equations of motion

For solving the given problem a vehicle model of Fig. 3 with a four wheel drive has been used. The model has 14 degrees of freedom. It includes all three basic subsystems of the vehicle.

Independent coordinates of the motion of the model can be arranged according to individual subsystems into a column matrix

$$q = [q_x^T \quad q_y^T \quad q_\varphi^T] \quad (3)$$

where

$$q_x = [x_B]$$

$$q_y = [y_B \quad \varphi_{Bx} \quad \varphi_{Bz} \quad y_{KFL} \quad y_{KFR} \quad y_{KRL} \quad y_{KRR}]^T$$

$$q_\varphi = [\varphi_{KFL} \quad \varphi_{KFR} \quad \varphi_{KRL} \quad \varphi_{KRR} \quad \varphi_2 \quad \varphi_1]^T$$

Equations of motion can be also derived by means of Lagrange equations of the 2nd kind. When using the model MIS-1 according to Misun (ref. 1 and 2) we obtain equations of motion without damping in matrix form

$$\begin{bmatrix} M_x & & \\ & M_y & \\ & & M_\varphi \end{bmatrix} \begin{bmatrix} \ddot{q}_x \\ \ddot{q}_y \\ \ddot{q}_\varphi \end{bmatrix} +$$

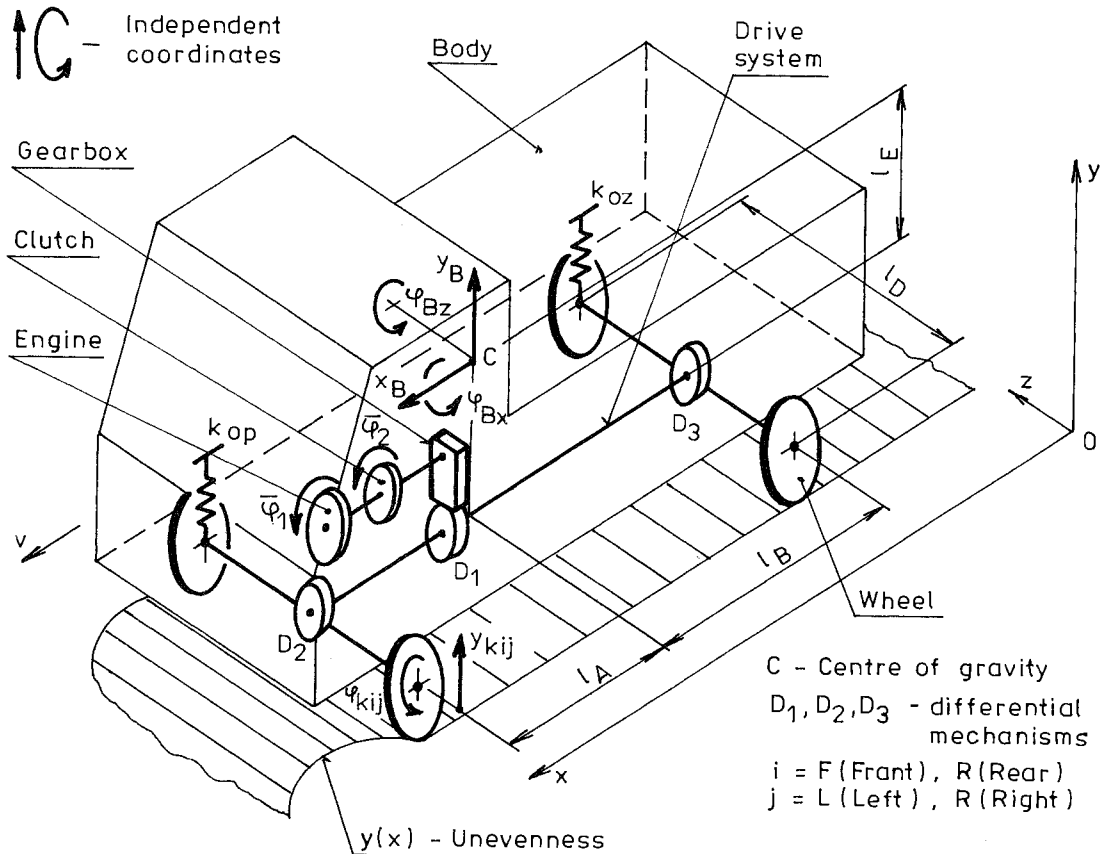


Fig. 3. Model of the vehicle and its driving subsystem

$$\begin{bmatrix} K_x & K_{xy} & K_{x\varphi} \\ K_{yx} & K_y & K_{y\varphi} \\ K_{\varphi x} & K_{\varphi y} & K_\varphi \end{bmatrix} \begin{bmatrix} q_x \\ q_y \\ q_\varphi \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \\ f_\varphi \end{bmatrix} \quad (4)$$

The mass submatrices of M_x, M_y, M_φ are diagonal, because with the system, relationships through the mass parameters of the model do not occur.

When running the vehicle wheel onto an unevenness of the road surface, all three subsystems of the model get into mutual relationships. These relationships are expressed by non-zero outside diagonal stiffness submatrices. E.g. a submatrix $K_{x\varphi}$ expresses a relationship between the subsystem of the whole model, which is moving in the direction of axes x, and the driving subsystem. The stiffness matrix K of the whole vehicle model is symmetrical.

The individual stiffness submatrices have then following forms at the position x_{ij} of the wheel on the road (i = F - front, R - rear; j = L - left, R - right) :

$$K_x = \sum c_{xij} \quad (5)$$

$$K_{xy} = [0 \quad 0 \quad -\sum c_{xij} (l_E + R_{xij}) \quad 0 \quad 0 \quad 0 \quad 0] \quad (6)$$

$$K_{x\varphi} = [a_{FL} \quad a_{FR} \quad a_{RL} \quad a_{RR} \quad 0 \quad 0] \quad (7)$$

where

$$a_{ij} = -c_{xij} \cdot R_{xij}$$

Stiffness submatrix K_y is symmetrical and has elements (upper triangle) :

$$K_y(1,1) = 2(k_{op} + k_{oz})$$

$$K_y(1,3) = 2(k_{op} l_A - k_{oz} l_B)$$

$$K_y(1,4) = K_y(1,5) = -k_{op}$$

$$K_y(1,6) = K_y(1,7) = -k_{oz}$$

$$K_y(2,2) = 0.5 l_D^2 (k_{op} + k_{oz})$$

$$K_y(2,4) = -K_y(2,5) = -0.5 l_D k_{op}$$

$$K_y(2,6) = -K_y(2,7) = -0.5 l_R k_{oz}$$

$$K_y(3,3) = 2(k_{op} l_A^2 + k_{oz} l_B^2) + \sum c_{xij} (l_E + R_{xij})$$

$$K_y(3,4) = K_y(3,5) = k_{op} l_A$$

$$K_y(3,6) = K_y(3,7) = -k_{oz} l_B$$

$$K_y(4,4) = c_{yFL} + k_{op}$$

$$K_y(5,5) = c_{yFR} + k_{op}$$

$$K_y(6,6) = c_{yRL} + k_{oz}$$

$$K_y(7,7) = c_{yRR} + k_{oz} \quad (8)$$

Submatrix $K_{y\varphi}$ in the similar way

$$K_{y\varphi}(3,1) = c_{xFL} R_{xFL} (l_E + R_{xFL})$$

$$K_{y\varphi}(3,2) = c_{xFR} R_{xFR} (l_E + R_{xFR})$$

$$K_{y\varphi}(3,3) = c_{xRL} R_{xRL} (l_E + R_{xRL})$$

$$K_{y\varphi}(3,4) = c_{xRR} R_{xRR} (l_E + R_{xRR})$$

$$K_{y\varphi}(4,1) = -c_{yFL} R_{yFL}$$

$$K_{y\varphi}(5,2) = -c_{yFR} R_{yFR}$$

$$K_{y\varphi}(6,3) = -c_{yRL} R_{yRL}$$

$$K_{y\varphi}(7,4) = -c_{yRR} R_{yRR} \quad (9)$$

The stiffness submatrix for the driving subsystem consist of two parts

$$K_\varphi = K_{\varphi 1} + K_T \quad (10)$$

when the stiffness matrix of the tyre itself is diagonal with elements

$$K_{\varphi 1} = \text{diag}(d_{FL} \quad d_{FR} \quad d_{RL} \quad d_{RR} \quad 0 \quad 0) \quad (11)$$

where (i = F,R ; j = L,R)

$$d_{ij} = c_{xij} R_{xij}^2 + c_{yij} R_{yij}^2$$

The stiffness matrix K_T of the driving system itself will be derived in sec. 2.2. The right sides of the equations (4) have following forms :

$$f_x = -\sum c_{xij} x_e \quad (12)$$

$$f_y = [-m_0 g \quad 0 \quad e_1 \quad e_{FL} \quad e_{FR} \quad e_{RL} \quad e_{RR}]^T \quad (13)$$

where are

$$e_1 = \sum c_{xij} (l_E + R_{xij}) \cdot x_e$$

$$e_{ij} = -m_K g + c_{yij} (y_{elj} - R_{yij} \varphi_{k0})$$

and

$$\varphi_{k0} = v_0 \cdot t / R \quad (14)$$

which expresses an uniform rotations of the wheels as a consequence of a moving road at the speed v_0 .

Further is

$$\mathbf{f}_\varphi = [p_{FL} \ p_{FR} \ p_{RL} \ p_{RR} \ 0 \ 0]^T \quad (15)$$

where

$$p_{ij} = c_{xij} R_{xij} x_e + c_{yij} R_{yij} (R_{yij} \varphi_{k0} - y_{eij})$$

2.2. Torsion driving subsystem

When locking some differential mechanisms (we introduce the marking $D = 0$ - unlocked, $D = 1$ - locked), the structure of the driving subsystem is changed, as its number of degrees of freedom is changed too. This locking of the differential mechanisms must be respected in order to receive more reliable dynamic properties of the driving subsystem.

For solving this problem a method (ref. 4) will be used which utilizes besides independent coordinates arranged into a vector

$$\mathbf{q}_\varphi = [\varphi_{KFL} \ \varphi_{KFR} \ \varphi_{KRL} \ \varphi_{KRR} \ \varphi_2 \ \varphi_1]^T \quad (16)$$

also dependent coordinates of the driving subsystem (see Fig. 4)

$$\mathbf{q}_{\varphi D} = [\varphi_{D1} \ \varphi_{D2} \ \dots \ \varphi_{D9}]^T \quad (17)$$

The driving subsystem in Fig. 4 is reduced to the half-axels of the wheels.

The motion of this model is described according to

- matrix equation of motion

$$\mathbf{M}_\varphi \ddot{\mathbf{q}}_\varphi + \mathbf{K}_1 \mathbf{q}_\varphi + \mathbf{K}_2 \mathbf{q}_{\varphi D} = \mathbf{0} \quad (18)$$

- matrix algebraic equation

$$\mathbf{R} \mathbf{q}_{\varphi D} + \mathbf{S} \mathbf{q}_\varphi = \mathbf{0} \quad (19)$$

which expresses kinematic and force relations of all three differential mechanisms. The elements of matrices \mathbf{R} , \mathbf{S} are changed with a change of the locking of the individual differential mechanisms.

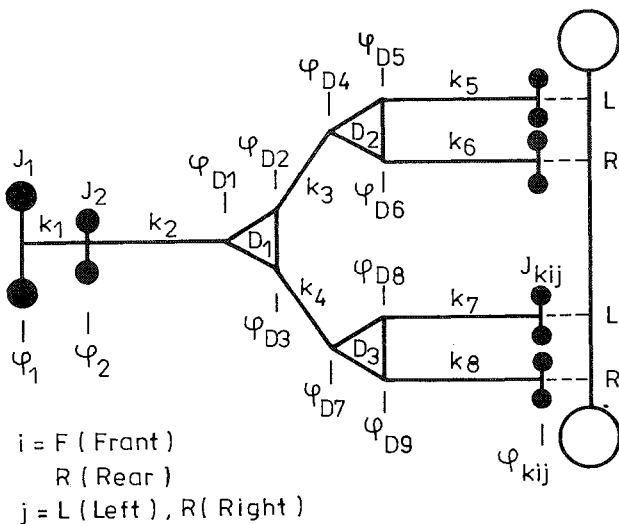


Fig. 4. Model of the driving subsystem

From the relation (19) we evaluate

$$\mathbf{q}_{\varphi D} = -\mathbf{R}^{-1} \mathbf{S} \mathbf{q}_\varphi \quad (20)$$

as the matrix \mathbf{R} is regular.

After substituting the relation (20) into (18) and rearranging we obtain equations of motion in independent coordinates only

$$\mathbf{M}_\varphi \ddot{\mathbf{q}}_\varphi + \mathbf{K}_\varphi \mathbf{q}_\varphi = \mathbf{0} \quad (21)$$

where the stiffness matrix of the driving subsystem is

$$\mathbf{K}_\varphi = \mathbf{K}_1 - \mathbf{K}_2^{-1} \mathbf{R} \mathbf{S} \quad (22)$$

After substituting in the relations (10) and (4) we obtain equations of motion of the whole vehicle model in total independent coordinates

$$\mathbf{M} \ddot{\mathbf{q}} + \mathbf{K} \mathbf{q} = \mathbf{f} \quad (23)$$

2.3. Spectral and modal properties of a vehicle model

Among the fundamental dynamic characteristics of conservative mechanical systems are their spectral and modal properties.

To the matrices \mathbf{M} , \mathbf{K} in the relation (23) we evaluate:

- spectral matrix $\lambda = \text{diag}\{\omega_{0i}^2\} = \text{diag}\{(2\pi f_{0i})^2\}$

for $i = 1, 2, \dots, 14$

- modal matrix $\mathbf{V} = [\mathbf{v}_1 \ \mathbf{v}_2 \ \dots \ \mathbf{v}_{14}]$

Table 1 shows the natural frequencies f_{0i} of the model and Table 2 corresponding shapes \mathbf{v}_{0i} of vibrations when the model is driving on a horizontal even road, i.e. at $\alpha = 0$.

The basic parameters of the model are:

$m_B = 20000 \text{ kg}$, $m_K = 448 \text{ kg}$, $l_A = 0.6 \text{ m}$

$l_B = 1.4 \text{ m}$, $l_E = 0.5 \text{ m}$, $l_D = 1.8 \text{ m}$

$k_{op} = 141000 \text{ N/m}$, $k_{oz} = 141000 \text{ N/m}$

From Table 1 an important fact follows that the spectral and modal properties of the model are changed when locking some of the differential mechanisms. When all three differentials are unlocked (state 000) then three natural frequencies are identical, i.e.

$$f_{07} = f_{08} = f_{09} = 5.29 \text{ Hz}$$

while the corresponding shapes of the vibrations are different. This is due to the internal degrees of freedom of the driving subsystem. By these three shapes of vibrations the driving subsystem is not under load because there are the torsion vibrations of the vehicle wheels only.

When all three differentials are locked (state 111), the system does not contain any internal degrees of freedom any more, the three natural frequencies of the torsional subsystem f_{07} , f_{08} , f_{09} vary and therefore the driving subsystem is under loading at each of these three shapes of vibrations.

Table 1. Natural frequencies of the model, $m_0 = 20000$ kg

Diff. state $D_1 D_2 D_3$	P	Natural frequencies f_{oi} [Hz]													
		f_{01}	f_{02}	f_{03}	f_{04}	f_{05}	f_{06}	f_{07}	f_{08}	f_{09}	f_{010}	f_{011}	f_{012}	f_{013}	f_{014}
000	1	0	0.50	0.79	1.6	1.80	5.29			7.4	8.71	8.72	8.73	64.53	
	3		0.68	1.01	1.8	1.95	5.29			7.5				46.11	
	5		0.71	1.19	1.8	3.66	5.29			8.2				47.71	
100	3	0	0.68	1.02	1.8	1.97	5.29		6.9	8.3	8.71	8.72	8.73	46.18	
011				1.01		1.95	5.3	7.5	8.3	8.3				46.11	
111				1.02		1.98	6.9	8.3	8.3	8.4				46.18	

Table 2. The shapes of vibrations, $m_0 = 20000$ kg, $P = 3$, $D_1 D_2 D_3 = 000$, $\alpha = 0$

Coordinates	Natural frequencies f_{oi} [Hz]													
	f_{01}	f_{02}	f_{03}	f_{04}	f_{05}	f_{06}	f_{07}	f_{08}	f_{09}	f_{010}	f_{011}	f_{012}	f_{013}	f_{014}
	0	0.68	1.01	1.80	1.95	5.29			7.48	8.71	8.72	8.73	46.1	
	Shapes of vibrations v_i													
	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}
x_B	.21	-.05	-.06	0.0	-.07	0.0	0.0	0.0	-.01	0.0	0.0	0.0	0.0	0.0
y_B		.30	-.11	0.0	.02				0.0	0.0	-.01	-.01	0.0	
ϕ_{Bx}		0.0	0.0	.98	0.0				0.0	0.0	0.0	0.0	-.03	
ϕ_{Bz}		-.21	-.17	0.0	.25				.01	0.0	.01	-.01	0.0	
YKFL	.0	.04	-.01	.10	-.02	0.0	0.0	0.0	-.01	-.50	.70	-.15	.50	0.0
YKFR		.04	-.01	-.10	-.02				-.01	.50	.70	-.15	-.50	
YKRL		.01	-.04	.10	.04				.01	.50	.15	.70	.50	
YKRR		.01	-.04	-.10	.04				.01	-.50	.15	.70	-.50	
ϕ_{KFL}						-.41	-.71	-.29						
ϕ_{KFR}		.36	.35		-.02	-.41	.71	-.29	.50		.01	-.03		-.01
ϕ_{KRL}						.82	0.0	-.29						
ϕ_{KRR}	.40			0.0		0.0	0.0	.87		0.0			0.0	
ϕ_2		.41	.48		.67	0.0	0.0	0.0	-.03			.01		.995
ϕ_1		.41	.48		.69	0.0	0.0	0.0	-.04		0.0	.01		-.09

From Table 1 also follows a change of natural frequencies of the model when a different gear box ratio P ($P = 1, 3, 5$) is engaged. Some natural frequencies are substantially changed according to the engaged gear box ratio, e.g. f_{02}, f_{03}, f_{05} .

Table 2 shows the shapes of vibrations of the model at unlocked all three differential mechanisms and at $\alpha = 0$. The shapes of vibrations v_6, v_7, v_8 confirm the fact, that by these shapes the driving subsystem does not come under loading.

The individual shapes of vibrations clearly demonstrate the relationships between the individual basic subsystems of the vehicle model (they are separate by a dash line). For example the relationship of all three model subsystems is represented by the shape of vibrations v_2 , by which vertical vibrations of the model, its til-

ting, its forward vibrations and torsion vibrations of the driving subsystem occur simultaneously. On the contrary at the shape of vibrations v_4 the whole model vibrates vertically only and this motion is without any relationships to the other two subsystems.

3. TRAVERSING OF THE MODEL OVER A ROAD UNEVENNESS

When solving the response of the model according to Fig. 3 when traversing over an individual unevenness we shall introduce a damping matrix B which has a similar or simpler structure to a stiffness matrix K .

Then the equation of motion in matrix form is

$$M \ddot{q} + B \dot{q} + K q = f \tag{24}$$

The equation (24) is a non-linear differential equation, because the elements of matrices **B**, **K** change with a change of the angle α , which means they change with the position of the wheel on the road. When solving the response it is therefore necessary to know exactly the position of the model and the position of individual wheels on the road. According to these positions to the given unevenness, corresponding values

$$y_{elj}(x), R_{xij}(x), R_{yij}(x), \alpha_{ij}(x)$$

will be used, while for wheels of the MIS-1 model is a common variable

$$x_e = v_0 t \tag{25}$$

For appropriate solving of the response of the model Newmark numerical integrational method was used. In every integrational step it is necessary to carry out corrections of the elements of the damping matrix **B** and stiffness matrix **K** and, of course, of the elements of the vector **f** on the right side of the equation of motion (24).

After having solved the response of the model in independent coordinates $q = [q_x^T, q_y^T, q_\varphi^T]^T$ we fi-

nish the calculation of the response of the model in dependent coordinates according to the relation (20), thus

$$q_{\varphi D} = -R^{-1} S q_\varphi \tag{26}$$

Therefore we can evaluate the motion of any member of the model, its position, speed, and eventually loading or stress.

So, for example, the torque of the clutch shaft is given by the relation

$$T_c = k_1(\varphi_1 - \varphi_2)/i_c \tag{27}$$

where i_c is the total gear ratio between the clutch shaft and the drive half-axes.

In the similar way we can obtain the loading of the front right half-axle (see Fig. 4)

$$T_{KFR} = k_6(\varphi_{D6} - \varphi_{KFR}) \tag{28}$$

Thus it is possible to evaluate the loading of any nodal points of the driving subsystem.

In Fig. 5 and following figures, you can see

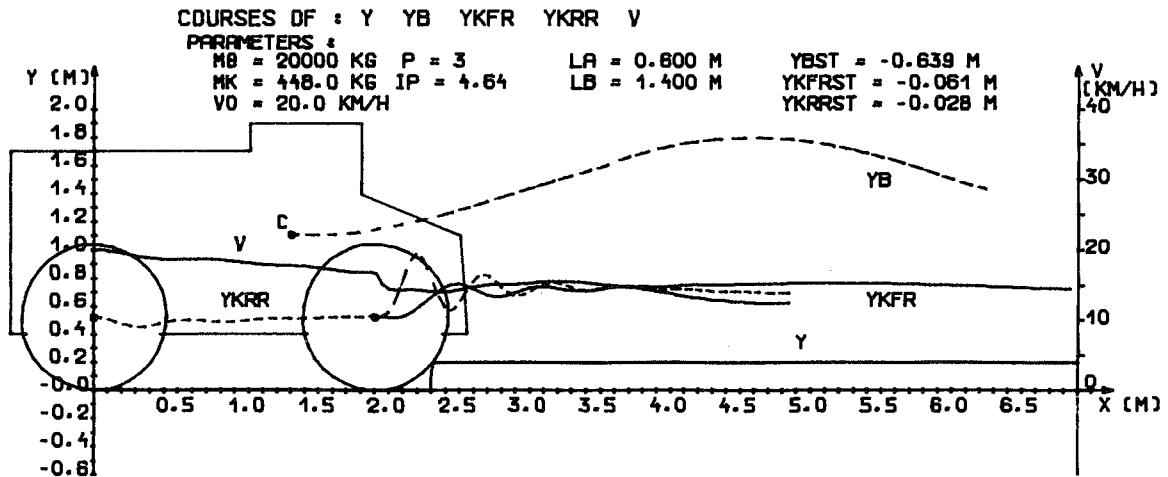


Fig. 5. Running of the model on the step, $v_0 = 20$ km/h, $P = 3$

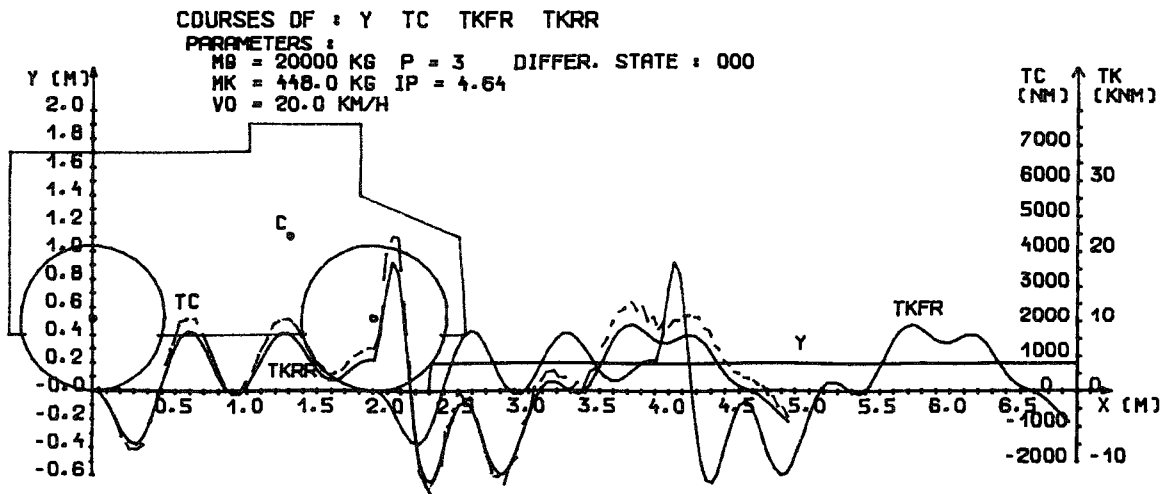


Fig. 6. Loading of the driving system, locking 000

several selected courses of the model responses. To the given unevenness $y(x)$ there are for example pictured the forward speed v of the model, the vertical wheel motions $y_{KFR}(x), y_{KRR}(x)$ in the right track, and the vertical motion of the centre of gravity $y_B(x)$ of the model. Or to the given unevenness $y(x)$, you can also see the courses of torque $T_C(x)$ on the clutch shaft, torques $T_{KFR}(x), T_{KRR}(x)$ on the front and rear half-axels.

4. THE EVALUATION OF RESULTS

The presented vehicle model enables one to observe its response when running over unevenness of the road surface of any shape with a possibility of choosing various parameters of the model (mass, stiffness, geometry) or various running conditions (initial speed, combination of locking the differential mechanisms, engaged gear box ratio).

From Fig. 5 and others we can say following conclusions :

- when crossing over an unevenness of the road surface the forward speed of the model changes and this change is not linear. It is a conse-

quence of different shapes of unevenness and also of the relationships among the subsystems of the model because when passing over an unevenness the torsion subsystem influences the forward motion of the model

- when passing over an unevenness, the wheel is excited both in the vertical and in the horizontal directions through a variables $y_e(t)$ and $x_e(t)$
- the circular shape of the model wheel causes the wheel to become excited by the unevenness before its centre comes just over the beginning of the unevenness
- the driving subsystem of the model is under substantial loading when passing over individual unevenness
- the introducing of the forces of the gravity of individual members of the model plays a significant role on the response of the model and thus on the loading and stress of the driving subsystem, too
- the response of the model including the stress of the driving subsystem changes both with the shape of the unevenness passed and with the initial speed of the model
- at the on-running of the front wheels onto the

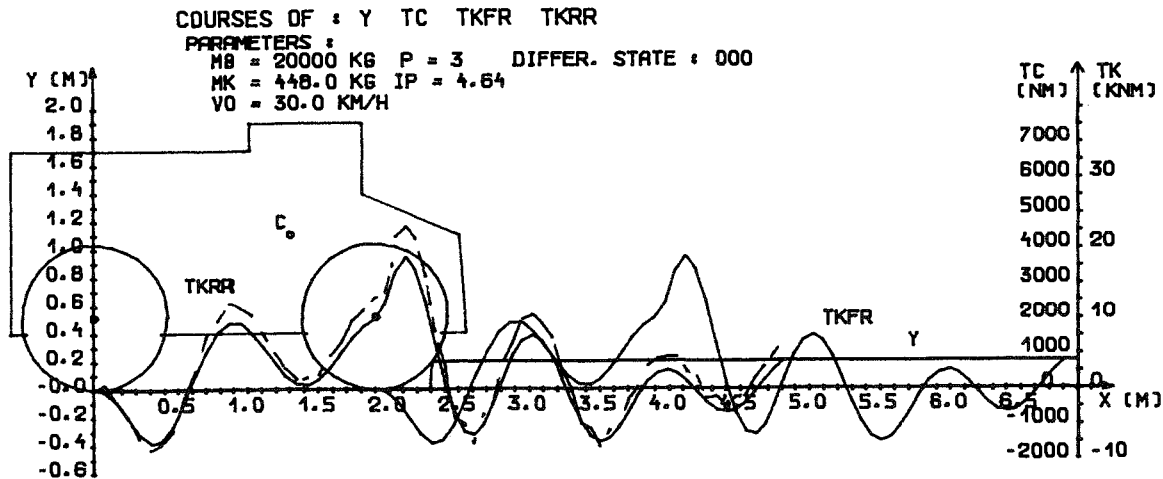


Fig. 7. Loading of the driving system, locking 000

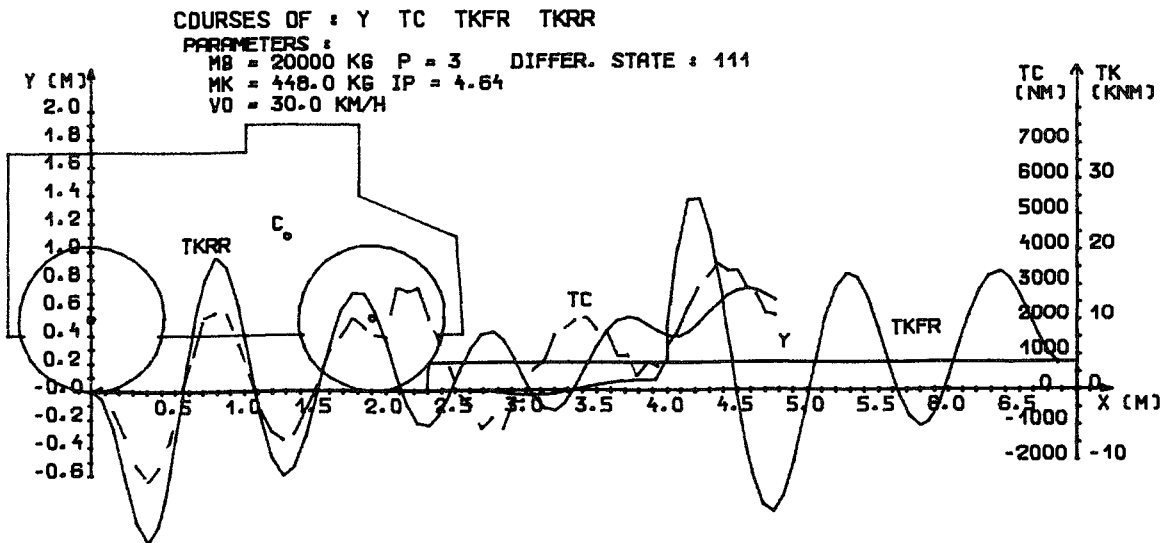


Fig. 8. Loading of the driving system, locking 111

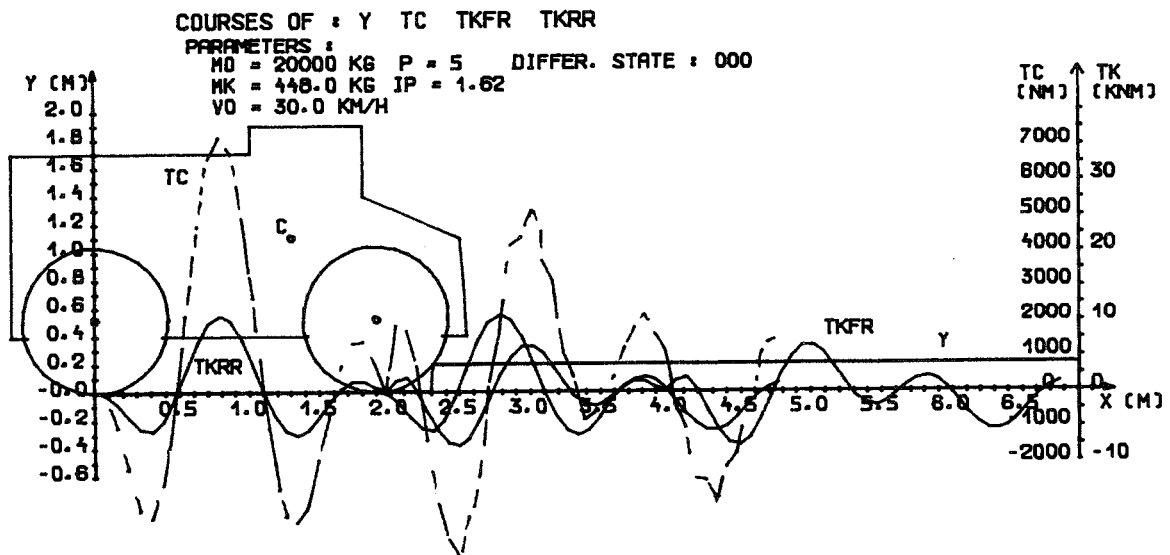


Fig. 9. Loading of the driving system, $P = 5$, locking 000

unevenness, the whole driving subsystem, i.e. the clutch shaft and rear half-axels, comes under stress

- the locking of the differential mechanisms influences the magnitude of torques which put the driving subsystem under stress. When all differentials mechanisms are locked, the torques on individual half-axels differ one from another
- the driving subsystem is under a higher stress when a higher gear box ratio is engaged than with a lower one

5. CONCLUSION

The model of a vehicle presented with two driven axels takes in consideration a number of important parameters of real vehicles. Among the important parameters are for example the circular shape of the wheel with a yielding tyre, forces of gravity of individual members of the model, locking of the differential mechanisms and very important relationships amongst individual subsystems when passing over an unevenness of the road surface.

These relationships are of varying significance according to the position of the wheel on the unevenness, i.e. they vary with changes of the angle α .

With changes of these relationships, the spectral and modal properties and also the dynamic properties of the entire model are changed, too. When the wheel is positioned on the unevenness, mutual influencing of all three basic subsystems of the model takes place. Thus for example, the torsion motion of the driving subsystem influences the change of the forward speed of the model and vice versa.

The aim of the contribution is to solve the loading of the driving subsystem when passing over an unevenness of the road surface. That means that the respecting of the above mentioned relationships is very desirable and necessary. The model enables the examinations of the loading and stress of the vehicle driving sub-

system at various model parameters, at various operational conditions and at various shapes of road unevenness.

The author of the paper dealt also with other exciting effects from the side of traversing unevenness and their influence on the loading and stress of vehicle drives. For example he dealt with the excitation of the torsion subsystem of the drive of TATRA lorries, due to the swinging of their half-axels when running over unevenness of the road surface.

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