ESTIMATION OF UNKNOWN INPUTS OF HEAVY VEHICLE: EXPERIMENTAL VALIDATION

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Abstract: In this work, a method is developed to estimate the unknown inputs of heavy vehicle. These inputs represent road profile which is very interesting since it is used to estimate the vertical forces acting on the wheels. These unknown inputs are reconstructed using higher order sliding mode observer. First, speeds and accelerations of heavy vehicle are estimated in finite time. Validation process is done using an instrumented heavy vehicle and measured road profile by LPA instrument. Keywords: Heavy vehicle, Road profile, High order sliding mode observers.

1 Introduction

Many researches on vehicle-road interaction are performed without taking into account the response of the road profile. Recently, the responses of the road profile to dynamics vehicle forces have been investigated. However, these inputs are not well known and there are some difficulties to have them in real time. Then, it’s not easy to estimate the vertical forces on board.

A variety of sensors and instruments can be used for measuring these inputs (LPA, inertial method, profilometers...). However, the sensors are expensive and the measurements are not done on board.

In this work, we propose a new method to evaluate the road profile inputs based on higher order sliding mode observers which are considered as inputs of the developed heavy vehicle model.
First, these inputs are measured by LPA instrument (Longitudinal Profile Analyzer), APL in French, developed at Roads and Bridges Central Laboratory (LCPC in French) and applied to the heavy vehicle model ([7], [4]). Then a third order sliding mode observer is developed to estimate these unknown inputs ([1]). This estimation allow us to reconstruct the vertical forces which are very important to calculate road damage or to evaluate the risk of rollover of the heavy vehicle using the Load Transfer Ratio (see ([5], [6])).

In a validation procedure, we compare the LPA measures with simulation results coming from a developed estimator. We compare also the states and vertical forces estimations with those given by an instrumented tractor of Renault Trucks. The details of this instrumentation is given in this paper.

This paper is organized as follows: section 2 deals with the heavy vehicle model description. The design of observer to estimate the road profile and vertical forces is presented in section 3. Section 4 gives some experimental results related to the estimation in order to show robustness of a proposed method. Finally, some remarks and perspectives are given in the concluding section.

2 Heavy vehicle modelling

Many studies deal with heavy vehicle modelling which represent a complex mechanical system with nonlinear features. The models used for heavy vehicles are very complicated ([2], [3]). Consequently, it is relatively difficult to define the different parameters of these models. In this paper, we consider the tractor model with 5 degrees of freedom (dof) composed of two principal bodies: the first, is the unsprung part of the vehicle (unsprung mass, four wheels and front axle), and the second body represents the sprung mass. All the dynamic parameters of the model represented in the figure (1), are considered as known. Some of them are given by the constructor and others are measured during the tests described in this paper.

![Figure 1: Instrumented tractor](image-url)
This model is derived using Lagrangian’s equations.
We can define a dynamic model of the vehicle as:

\[ M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + K(q) = F_g \]  

where \( M \in \mathbb{R}^{5 \times 5} \) is the inertia matrix (Mass matrix), \( C \in \mathbb{R}^{5 \times 5} \) is related to the damping effects, \( K \in \mathbb{R}^5 \) is the springs stiffness vector and \( F_g \in \mathbb{R}^5 \) is a generalized forces vector.

\( q \in \mathbb{R}^5 \) is the coordinates vector defined by:

\[ q = [q_1, q_2, q_3, q_4, \theta]^T \]  

The vertical acceleration of the chassis (tractor’s body) is obtained as following:

\[ \ddot{z} = (K_1 q_1 + K_2 q_2 + (K_1 - K_2) \frac{T_w}{2} \sin(\theta) + B_1 \dot{q}_1 
+ B_2 \dot{q}_2 - (B_1 - B_2) \frac{T_w}{2} \cos(\theta) \dot{\theta})/M \]  

\( \theta \) is the tractor roll angle;
\( q_1, q_2 \), are respectively the left and right front suspension deflection of the tractor;
\( q_3, q_4 \), are respectively the left and right rear suspension deflection of the tractor;
\( z \) is the vertical displacement of the tractor sprung mass (center of gravity height) and \( \ddot{z} \) is the vertical acceleration of the sprung mass.

The suspension is modeled as the combination of spring and damper elements as shown in the figure (2).

The tractor chassis (sprung mass) with the mass \( M \) is suspended on its axles through two suspension systems. We can also show that the tire is modeled by springs with the coefficients \( k_i \) and damper elements with the coefficients \( B_i \). The wheel masses are represented by \( M_i \). At the tire contact, the road profile is represented by the inputs \( u_i \), which are considered as the heavy vehicle inputs.

The vertical displacement of the wheel \( i \) is represented by \( z_{ri}, i=1..4 \). The vertical displacement of the front wheels can be calculated using the following equations:

\[
\begin{cases}
  z_{r1} = z - (q_0 + q_1) + \frac{T_w}{2} \sin(\phi) - r \\
  z_{r2} = z - (q_0 + q_2) - \frac{T_w}{2} \sin(\phi) - r
\end{cases}
\]  

where \( q_0 \) is the static distance between the COG and the axles of the vehicle, \( T_w \) is the tractor track width and \( r \) is the wheel’s radius.

We obtain the vertical accelerations of the front wheels using the following equations:

\[
\begin{cases}
  \ddot{z}_{r1} = (B_1 \dot{q}_1 + K_1 \frac{T_w}{2} \sin(\phi) + B_1 \frac{T_w}{2} \cos(\phi) \dot{\phi} 
+ K_1 q_1 - k_1 z_{r1} + k_1 u_1)/m_1 \\
  \ddot{z}_{r2} = (B_2 \dot{q}_2 - K_2 \frac{T_w}{2} \sin(\phi) - B_2 \frac{T_w}{2} \cos(\phi) \dot{\phi} 
+ K_2 q_2 - k_2 z_{r2} + k_2 u_2)/m_2
\end{cases}
\]  

Using these equations, the unknown inputs acting on the two front wheels can the
be written as following:

\[
\begin{align*}
  u_1 &= (m_1 \ddot{z}_{r1} - B_1 \dot{q}_1 + K_1 \frac{T_w}{2} \sin(\theta) \\
         &+ B_1 \frac{T_w}{2} \cos(\theta) \dot{\theta} - K_1 q_1 + k_1 z_{r1})/k_1 \\
  u_2 &= (m_2 \ddot{z}_{r2} - B_2 \dot{q}_2 - K_2 \frac{T_w}{2} \sin(\theta) \\
         &- B_2 \frac{T_w}{2} \cos(\theta) \dot{\theta} - K_2 q_2 + k_2 z_{r2})/k_2
\end{align*}
\]  

(6)

The normal forces \( F_{ni} \) \((i = 1, \cdots, 4)\) acting on the front wheels are calculated using the following expression:

\[
F_{ni} = F_{ci} + k_i (u_i - z_{ri}), \ i = 1, \cdots, 4
\]  

(7)

where \( F_{ci} \) is the static force due to the static mass of the vehicle.

We assume that the force generated by damping effect is neglected compared to the spring force \( k_i (u_i - z_{ri}) \).

The next section deals with the development of the high order sliding mode observer to estimate all states of the system, to reconstruct the unknown inputs, namely, road profile and to deduce the vertical forces of the heavy vehicle.

3 Estimation of the unknown inputs

In this section, we develop the third order sliding mode observer to estimate the heavy vehicle dynamic states and the road profile which allow to reconstruct the vertical forces [1].

In order to develop the observer, we rewrite the dynamic model defined in (1) in the state form as follows:

\[
\begin{align*}
  \dot{x}_1 &= x_2 \\
  \dot{x}_2 &= x_3 = M^{-1}(F_g - C(x_1, x_2)x_2 - K(x_1)) \\
  y &= x_1
\end{align*}
\]  

(8)
where the state vector \( x = (x_1, x_2)^T = (q, \dot{q})^T \), and \( y = q \) is the measured outputs vector of the system.

Thus, we obtain:

Before developing the sliding mode observer, let us consider the following assumptions (see [1]):

1. The state is bounded (\( \|x(t)\| < \infty, \forall \ t \geq 0 \)).
2. The system inputs are bounded (\( \exists \) a constant \( \mu \in \mathbb{R} \) such as: |\( u_i \)| < \( \mu \), \( i = 1..5 \)).
3. The generalized forces \( F_g \) are bounded (\( \exists \) a constant \( \zeta \in \mathbb{R} \) such as: \( F_{gi} < \zeta \), \( i = 1..5 \)).
4. In the vector \( C(x_1, x_2)x_2 \), we find quadratics elements in \( x_2 \). We can then bound this vector as:

\[
\| C(x_1, x_2)x_2 \| \leq c_1 \| x_2 \|^2
\]  

(9)

This last assumption is coming from mechanical and physical properties of our system and because of boundness of the real signals (position, speed, acceleration).

We propose in this work, the following third order sliding mode observer [1]:

\[
\begin{align*}
\dot{x}_1 &= x_2 - \lambda_0 |x_1 - x_1|^{2/3} \text{sign}(x_1 - x_1) \\
\dot{x}_2 &= \dot{x}_3 - \lambda_1 |\dot{x}_2 - \dot{x}_1|^{1/2} \text{sign}(\dot{x}_2 - \dot{x}_1) \\
\dot{x}_3 &= -\lambda_2 \text{sign}(x_3 - \dot{x}_2)
\end{align*}
\]

(10)

where \( \dot{x}_1, \dot{x}_2 \) and \( \dot{x}_3 \) are respectively the estimate of \( x_1, x_2 \) and \( \ddot{x}_2 \).

This observer permit to estimate both positions, speeds and accelerations of the system. \( \lambda_0, \lambda_1 \) and \( \lambda_2 \in \mathbb{R}^5 \) are the observer gains.

The estimation errors are obtained using the equations (8) and (10) as following:

\[
\begin{align*}
\dot{\hat{x}}_1 &= x_2 - \dot{x}_2 + \lambda_0 |x_1 - x_1|^{2/3} \text{sign}(x_1 - x_1) \\
\dot{\hat{x}}_2 &= \dot{x}_3 - \dot{x}_2 + \lambda_1 |\dot{x}_2 - \dot{x}_1|^{1/2} \text{sign}(\dot{x}_2 - \dot{x}_1) \\
\dot{\hat{x}}_3 &= \dot{x}_2 + \lambda_2 \text{sign}(x_3 - \dot{x}_2)
\end{align*}
\]

(11)

where \( \hat{x}_i = x_i - \dot{x}_i \ (i = 1, \ldots, 3) \) is the estimation error of the variable \( x_i \).

The Jerk of the system is bounded, satisfying the inequality:

\[
f^+ \geq 2 \| \ddot{y} \|
\]

(12)

Chosen the \( i \)th components of \( \lambda_0, \lambda_1 \) and \( \lambda_2 \) as: \( \lambda_0^i = 3\sqrt{f^+}, \lambda_1^i = 15\sqrt{f^+}, \lambda_2^i = 1.1 f^+, i = 1..5 \), we obtain the convergence in finite time \( t_0 \), both the positions, speeds and accelerations \( x_1, x_2 \) and \( x_3 \) (see [1]).

Let now consider the convergence of the acceleration of the body for \( t > t_0 \).

Let now consider the convergence of the acceleration of the body for \( t > t_0 \). According to the equation 3, and since all the coordinates vector \( q \) and its derivative \( \dot{q} \), are estimated, we can calculate this acceleration as follows:

\[
\ddot{q} = (K_1 \dot{q}_1 + K_2 \dot{q}_2 + (K_1 - K_2) \frac{T\dot{w}}{2} \sin(\dot{\theta}) + B_1 \dot{q}_1) + B_2 \dot{q}_2 - (B_1 - B_2) \frac{T\dot{w}}{2} \cos(\dot{\theta}) \dot{\theta}) / M
\]

(13)
where \(^{\wedge}\) represents the estimated symbol.

The estimation error is obtained from (3) and (13) as follows:

\[
\ddot{\hat{z}} = (K_1 \hat{q}_1 + K_2 \hat{q}_2 + (K_1 - K_2) \frac{T_w}{2} \sin(\hat{\theta}) + B_1 \dot{\hat{q}}_1 \\
+ B_2 \dot{\hat{q}}_2 - (B_1 - B_2) \frac{T_w}{2} (\cos(\hat{\theta}) \dot{\hat{\theta}} - \cos(\hat{\theta}) \dot{\hat{\theta}})) / M
\]

where \(^{\sim}\) represents the estimated error symbol. We can easily remark, related to the equation (14) and after convergence of \(x_1, x_2\) and \(x_3\), that the estimation error \(\ddot{\hat{z}}\) converges toward 0 in finite time \(t > t_0\).

Assuming that the initial conditions of speeds and positions are known, we are able to do a double integration to the acceleration \(\ddot{\hat{z}}\) in order to obtain the vertical displacement \(\hat{z}\) of the chassis.

On another hand, and from the equations 4, the vertical displacements of the wheels are estimated using the following equations:

\[
\begin{align*}
\ddot{z}_{r1} &= \ddot{\hat{z}} - \dot{\hat{q}}_1 + \frac{T_w}{2} \sin(\hat{\theta}) - r \\
\ddot{z}_{r2} &= \ddot{\hat{z}} - \dot{\hat{q}}_2 - \frac{T_w}{2} \sin(\hat{\theta}) - r
\end{align*}
\]

At time \(t > t_0\), all positions including the vertical displacement \(\hat{z}\) of the chassis and their first and second derivatives are estimated. In such case, the estimation of the road profile (unknown inputs) is possible. It’s calculated using the following equation:

\[
\begin{align*}
\dot{u}_1 &= (m_1 \ddot{z}_{r1} - B_1 \dot{\hat{q}}_1 + K_1 \frac{T_w}{2} \sin(\hat{\theta}) \\
&+ B_1 \frac{T_w}{2} \cos(\hat{\theta}) \dot{\hat{\theta}} - K_1 \hat{q}_1 + k_1 \dot{\hat{z}}_{r1}) / k_1 \\
\dot{u}_2 &= (m_2 \ddot{z}_{r2} - B_2 \dot{\hat{q}}_2 - K_2 \frac{T_w}{2} \sin(\hat{\theta}) \\
&- B_2 \frac{T_w}{2} \cos(\hat{\theta}) \dot{\hat{\theta}} - K_2 \hat{q}_2 + k_2 \dot{\hat{z}}_{r2}) / k_2
\end{align*}
\]

Let consider now, the equation 16. We can remark that since all states are now known and since the vertical accelerations of the wheels are measured , we can calculate the unknown inputs \(\dot{u}_1\) and \(\dot{u}_2\) in finite time.

The finite time estimation of the road profile and the vertical displacement of the wheels, allow finally to reconstruct the vertical forces in finite time using the following equation:

\[
\hat{F}_{ni} = F_{ci} + k_i (\dot{u}_i - \dot{\hat{z}}_{ri}), \quad i = 1 \ldots 4
\]

In the next section, we give some experimental results to show the quality of road profile estimation.
4 Experimentation

In this section some results related to the estimations of the states, the road profile and the vertical forces, in order to verify the robustness of the proposed approach, are shown.

4.1 Description

Several road profile with different amplitudes and frequencies are then measured by the Longitudinal Profile Analyzer (LPA) to excite the system.

The LPA instrument is represented in the figure (3).

![Figure 3: Longitudinal Profile Analyzer](image)

The figure (4) shows an example of the left and right road profile measured by this instrument.

In order to validate theoretical study and the simulations results, we instrumented the Truck of "Renault Trucks" described in the figure 1.

The vehicle is equipped with several sensors to measure the dynamics of the vehicle such as the angular speeds, accelerations, and the suspension deflections.

This study is done on behalf of French project, called VIF [9].

The figure 5 illustrates the used sensors:

- four sensors (number 4), LVDT installed between the wheel and the chassis in order to measure the deflections of suspensions,
- four accelerometers (number 5) installed on the chassis to measure the vertical accelerations of wheels,
- three axial gyroscopes (number 6) installed on the chassis, to measure the angular speeds,
Figure 4: Road profile measures

Figure 5: The used sensors on the Vehicle
• two lasers (number 7) installed at the bottom of the chassis to measure the vertical distance of the vehicle. Note that this distance is not used in this work.

4.2 Experimental results

In this section, the result of Zig-zag test is shown. The figure (6) represents the steering angle. The vehicle speed during this test is shown in the figure (7).

![Figure 6: Steering angle](image)

![Figure 7: Vehicle speed](image)

First, let see if the states are well estimated. In the figure (8), the estimation of the suspension deflection is shown.
We remark that these positions are well and fast observed compared to the measures given by LVDT sensors. We can notice that at the times 15s, 30s and 45s, the suspension deflection increased up from 0.01m to 0.025m.

![Figure 8: Estimation of suspension deflections](image)

The roll angle is observed and represented in the figure (9). We can notice that this state converges in finite time to the measured one.

![Figure 9: Roll angle estimation](image)

Since the vertical acceleration of the body is well estimated, the double integration of this acceleration, gives an accurate estimation of the height center of gravity of the body compared to the true one. The result is shown in the figure (10).

As we said in the previous section, using the equation 15, and if we can estimate the positions vector \( q \) and the center height of gravity of the body \( z \), we can calculate the vertical displacements of the wheels \( z_{r1} \) and \( z_{r2} \). This estimation is then shown in the figure (11).
Figure 10: Estimation of centre height of gravity

Figure 11: Estimation of vertical displacements of the wheels
The road profile is calculated using the equation 6. Since at this stage, all the states are observed, we can deduce the road profile. This estimation is shown in the figure (12). We notice that these signals are estimated with small errors compared to those measured by the LPA instrument.

![Figure 12: Road profile estimation](image)

From the estimation of the vertical displacement of the wheels and the road profile, we can now reconstruct the vertical forces. The result is shown in the figure (13).

![Figure 13: Estimation of vertical force: left wheel](image)

At the top of this figure, the front vertical forces are shown and at the bottom, the rear vertical forces are presented. We remark that the vertical forces of the vehicle converge in finite time.

As the measured forces are not available during the tests, a direct comparison to
show the robustness of this estimation is then not possible. However, we remark the same phenomena than for the suspension deflections, since at the times 15s, 30s and 45s, the front left vertical force increased up to 40dN following then the measured suspension deflection.

5 Conclusion and perspectives

This paper proposes a method to estimate the unknown input under each wheel of heavy vehicle. This estimation permit to deduce the vertical forces. A third order sliding mode observer is then developed. The use of this observer is to be able to estimate in finite time all states of the system including accelerations. The setting of an estimator needs a model. In this work, we develop a tractor model which validated by sensors installed in the instrumented vehicle. In the second step of the work, the strong observability of the system is verified and the sliding mode observer is developed. Experimental results are done on la Valbonne site of Renault Trucks, in France, and showed that the suspension deflection, roll angle and its speeds and accelerations converge quickly and in finite time. These well estimations allow to reconstruct the acceleration of the height center of gravity of the body and the acceleration of the wheels. This permits also to estimate well and fast the vertical displacements of the wheels. The experimental results show an accurate reconstruction of the road profile since it's compared to the measures done by LPA instrument. Finally and from previous estimation, the presented result showed that the convergence of the vertical forces seems to be of quality.

The future work will concern the application of this method to the instrumented heavy vehicle (Tractor+semi-trailer) to estimate on board, the road profile and vertical forces under each wheel. Some dynamic parameters of the vehicle, like spring stiffness or the total mass of the vehicle, will be on-line identified.

References


