MEASUREMENT AND ANALYSIS OF THE DYNAMIC RESPONSE OF FLEXIBLE PAVEMENTS

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ABSTRACT

A method for calculating the transient response of road surfaces is presented. The method is based on the well-known convolution integral and as such assumes that the road is a linear isotropic structure. These assumptions are investigated and the model is validated by experiments on an instrumented track. The method is then used to predict theoretical road responses to variations in load speed and frequency.

The authors are grateful to the Director of the Transport and Road Research Laboratory and the members of the TRRL Pavement Design and Maintenance Division and the Vehicles and Environment Division for making available the test track and vehicle, and for assistance in carrying out the tests. Thanks also to David Cole for help in collecting and processing the vehicle data.

1. INTRODUCTION

Heavy vehicles travelling along irregular road surfaces generate dynamic tyre forces which in turn cause dynamic stresses and strains in the road structure. In order to examine the relationships between dynamic tyre forces and pavement deterioration it is necessary to determine the transient responses of roads as they are traversed by fluctuating tyre forces.

A variety of models have been used for analysis of the dynamic response of roads. A brief review of the literature may be found in [**1**]. Models vary considerably in complexity but are almost exclusively based on linear theory and few have been validated by comparison with field experiments.

2. THEORY

Typical dynamic variations in axle-height as a lorry drives along a road are 10mm [**2**]. The total deflection of the road surface under loads generated by a lorry are of the order of 0.5mm [**3**] and therefore the dynamic behaviour of the lorry and road may be uncoupled.

The response of a single input linear system to a time varying force (which is not a function of the system response) is given by the convolution integral:

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)P(\tau)\,d\tau \tag{1}$$

where $h(t)$ is the unit impulse response function relating the output to the input.

If the force is moving over an isotropic surface with constant velocity, the response of any point on the surface is given by the convolution integral:

$$y(\mathbf{x},t) = \int_{-\infty}^{\infty} h(\mathbf{x}-\mathbf{D}-V\tau\mathbf{i}, t-\tau)P(\tau)\,d\tau \tag{2}$$

where $\mathbf{x}$ is the position vector of the point at which the response is required
$\mathbf{D}$ is the position vector of the force at time $t=0$
$V\mathbf{i}$ is the constant velocity of the force
and $h(\mathbf{x},t)$ is the impulse response of the surface when the input and output are separated by the vector $\mathbf{x}$.

If the response is required at equally spaced points under the wheelpath of a vehicle then $\mathbf{x}$, $\mathbf{D}$, and $\mathbf{i}$ are parallel vectors and may be converted to scalars for the purpose of further analysis. The integral in Eq. 2 may then be approximated by a summation for evaluation in a digital computer:

$$y_{k,l} = \Delta\theta \sum_{i=1}^{n} \sum_{j=0}^{N_0/\eta} h_{\alpha(k-d_i)-\gamma l+\beta j,\eta j} P_{\nu l-\mu j,i} \tag{3}$$

where $y_{k,l} = y(k\Delta x, l\Delta t) = y(x,t)$
$h_{k,l} = h(k\Delta g, l\Delta\vartheta) = h(x,t)$
$P_{l,i} = P_i(l\Delta\tau) = P_i(t)$, the force applied by tyre i
Δx is the required space increment of the road response
Δt is the required time increment of the road response
Δg is the space increment of the road impulse response
$\Delta\vartheta$ is the time increment of the road impulse response
$\Delta\tau$ is the time increment of the force
$\Delta\theta$ is the time increment of the convolution calculation
n is the number of tyres
$d_i\Delta x = D_i$, the position of tyre i at time $t = 0$
$N_0\Delta\vartheta$ is the time by which the impulse response has decayed
$\alpha = \Delta x/\Delta g, \beta = V\Delta\theta/\Delta g$, $\gamma = V\Delta t/\Delta g$
$\eta = \Delta\theta/\Delta\vartheta$, $\nu = \Delta t/\Delta\tau$ and $\mu = \Delta\theta/\Delta\tau$

This expression may give the response at any point on the road. Sometimes, however, it is more convenient to calculate responses, $y(\hat{x}, t)$, at points moving with the vehicle (for instance beneath a tyre) and for this case Eq. 3 can be modified:

$$\hat{y}_{k,l} = \Delta\theta \sum_{i=1}^{n} \sum_{j=0}^{N_0/\eta} h_{\alpha(k-d_i)+\beta j,\eta j} P_{\nu l-\mu j,i} \tag{4}$$

where $\hat{y}_{k,l} = \hat{y}(k\Delta x, l\Delta t) = y(\hat{x}, t)$
and $\hat{x}$ is the response position in a reference frame moving with the vehicle.
i.e. $\hat{x} = x + Vt$.

As the suffices of h and P in Eqs. 3 and 4 are not necessarily integer, interpolation may be required. The accuracy of discrete convolution calculations depends on suitable choice of increments and also on the linearity and isotropy of the system being simulated.

3. IMPULSE RESPONSE MEASUREMENTS

Experimental measurements were conducted on two instrumented test track sections at the Transport and Road Research Laboratory (TRRL) to test the assumptions of the theory described above and to validate the results.

Test section A was constructed of 150mm of dense bituminous macadam (DBM), laid in two equal layers, with a wearing course of 50mm of hot rolled asphalt (HRA), built on 300mm of type 1* crushed granite subbase. The subgrade is a heavy clay with

* Type 1 specifies the content of a granular material by its proportion of particle sizes. Type 1 graded material is the standard grade for subbases in the UK.

CBR=2–3%. Test section B is built on a subgrade of hoggin with CBR≈3% and has a subbase of 225mm of type 1 crushed rock, a roadbase of 200mm of DBM, which was laid in two equal layers, and a wearing course of 50mm of HRA. Both of these sections are considerably weaker than typical motorway standards in the UK.

Each section has foil strain gauges bonded to the underside of the bituminous layers. The strains induced at this position in a road structure have frequently been associated with the fatigue-life of pavements.

Impulse responses were measured using an instrumented hammer and the strain gauges. The head of the hammer had a mass of 18.6kg and was mounted on a light, 2m long arm. The acceleration of the head was measured with an accelerometer and thus the force applied to the road during the impulse could be calculated. The hammer was designed so that the peak force would be around 40kN — a typical tyre load.

The outputs from the strain gauges were measured using AC bridge circuits and all of the data was logged digitally using a CED 1401 data-logger and a CED 1703 programmable filter/amplifier driven by an IBM PC/AT.

3.1 Linearity Tests

It is an assumption of the theory that the road behaves as a dynamically linear system. The literature is inconclusive on this point [**4,5,6**] so the test sections were tested for linearity. To investigate this property the hammer was dropped from three different heights, 2m, 1m and 0.5m. The applied impulse is proportional to the change in momentum of the head, which is in turn proportional to the square–root of the height from which the hammer is dropped:

$$I = m(1+e)\sqrt{2gh} \tag{5}$$

where I is the impulse
m is the mass of the hammer head
e is the coefficient of restitution
g is the acceleration due to gravity
and h is the height from which the hammer-head is released

For each hammer drop the impulse response function is obtained by dividing the Fourier transforms of the outputs by the Fourier transforms of the inputs and then inverse transforming the result [**7**].

The normalised impulse responses are shown in figures 1a and 1b. Figure 1a shows the response of section A when the hammer was dropped 1m away from the gauge and the gauge measured transverse (circumferential) strain. It is immediately obvious that the response from section A is linear: the normalisation causes the impulse response to be independent of the magnitude of the applied load. Figure 1b shows the response of section B when the hammer was dropped directly above the gauge. This section is stronger than section A and appears to give a less linear response. The normalised response is lower for smaller impulses. This is consistent with the characteristic "softening spring" behaviour of cohesive soils [**8**].

The reason for the difference in behaviour of the two sections is not clear and may depend on road temperature and moisture content in the subgrade. However, the nonlinearity observed in section B is only a 10% change for an impulse of doubled magnitude. The effect of nonlinearities on the convolution calculation is minimised by designing the hammer to load the road to levels similar to tyre forces [**9**].

3.2 Isotropy Tests

The convolution calculation relies on the assumption that road surface response is isotropic and hence the impulse response is not a function of the absolute position along the wheelpath but only of the separation between input and output. Both sections were tested for isotropy by dropping the impulse hammer at four different positions around the gauges. The normalised impulse responses from each test are shown in figures 2a and 2b.

Figure 2a shows the response of section A when the hammer was dropped 2m from the gauge. The two larger responses correspond to the gauge measuring longitudinal (radial) strain and the two smaller responses to transverse strain. There are small differences in corresponding responses indicating that section A may not be isotropic. Figure 2b shows the response of section B when the hammer was dropped 100mm from the gauge. The two larger responses now correspond to transverse strain and the two smaller ones to longitudinal strain and again there are differences indicating anisotropy, particularly in the longitudinal direction.

It should be noted that there was considerable difficulty in determining the exact location of the buried strain gauges. If the hammer blows were not equidistant from the gauges then no conclusions about the isotropy of the pavements may be drawn. Further investigation is required to determine the cause of the apparent anisotropy in these results.

It is common practice to assume isotropy in pavement response simulation. All static models of roads based on finite layer theory do this and for most purposes this provides an adequate tool for research. A few exceptions, however, may be found in the literature [**10,11**]. In these cases the road surface response is assumed to be locally isotropic, even though the properties are assumed to vary along the road.

3.3 Temperature Effects

Road responses vary significantly with environmental conditions and especially temperature. To monitor this effect the surface temperature of the road was measured as each impulse response was recorded. Figure 3 shows the peak of the normalised responses for each impulse on section A, taken with the hammer directly over the gauge, against the surface temperature. The normalisation procedure causes the impulse responses to be negative and hence the peaks are negative. There is clearly a correlation between the surface temperature and the peak responses but it should be noted that the surface temperature is not always indicative of the average temperature of the road structure. Figure 4 shows the daily variation of the temperature in section that was measured during the testing period with four thermocouples imbedded in the road structure. The diurnal variation of temperature in the subgrade is much less than that at the surface.

Because of the temperature sensitivity of the road response, the impulse responses that were used to calculate the road's response to a passing vehicle had to be measured just before the vehicle test was performed. These tests were carried out very early in the morning to minimise the effects of solar heating.

4. RESPONSE TO MOVING VEHICLE LOADS

4.1 Description of Experiments

A four-axle articulated lorry was instrumented so that its tyre forces could be logged by an on-board analogue tape-recorder [2]. The vehicle was driven over test section A on the TRRL test track in half-laden and fully-laden conditions at nominal speeds of 15,50 and 80km/h. In each test the lorry was driven over the measuring points with the outside tyres of the three sets of dual-tyres passing directly over the nominal position of the gauge. The single tyre on the steering axle was slightly in-board of the outside tyre of the dual-tyres. The location of the vehicle along the test section was monitored at three longitudinal positions by infra-red beams which put pulses on the recording tape corresponding to known positions on the road. Simultaneously, pulses were recorded by the roadside data-logger which also logged the pavement strains.

4.2 Details of the Calculation

The convolution calculation described by Eq. 3 was used to combine the measured wheel forces with the measured field of impulse responses. The time resolution for both the impulse responses and the digitised wheel forces was sufficiently high that interpolation between points in the time dimension was not required. However, impulse responses were only taken at 150mm intervals along the wheelpath and it was necessary to interpolate the impulse response field in the space dimension. No distinction was made for dual or single tyres and no correction was made for different wheel-paths. The effects of the offside tyre forces on the nearside road responses were considered negligible.

4.3 Results

Two typical results are shown in figures 5 and 6 which show the measured and predicted responses of road section A as functions of time. Each figure has four main peaks corresponding to passage of the four axles.

Figure 5 shows the transverse strain for a low speed (15km/h) test. Agreement between the predicted and measured responses is generally good except for the steering axle. This discrepancy is thought to be due to the lateral offset of the leading axle with its single tyre, which has a much narrower field of influence than a dual tyre. This, combined with errors in the nominal gauge position is likely to be the main source of the error.

Figure 6 shows the transverse strain for a high speed (80km/h) test. Again agreement is good although there are some errors. These are thought to be caused by:

(i) Lateral off-tracking of the tyres from the nominal gauge positions,
(ii) Inaccuracy in the exact location of the strain gauge,
(iii) Road surface temperature variations during the period between measurement of the impulse field and the lorry tests,
(iv) The influence of forces generated by the offside tyres,
(v) Dynamic contact area variations.

5. THEORETICAL ANALYSIS

Analysis of road responses measured in full-scale experimental tests is complicated by the dynamic axle loads of vehicles which increase with vehicle speed and are a function of the road roughness. The convolution calculation provides a convenient tool to examine road responses to idealised loads and hence to investigate road behaviour at a more fundamental level.

5.1 The Effect of Speed

The transverse strains induced in test section A by a constant 1kN load moving at different speeds were calculated. The convolutions were carried out using the moving frame formulation (Eq. 4) and the load was offset by 3m. The results are shown in figure 7.

The peak response falls rapidly as the load moves, falling by approximately 10% between static and 5m/s and another 10% between 5 and 20m/s. Above this speed the response changes less dramatically. This is consistent with other experimental evidence, see for example [**3**]. The response beneath the force continues to fall but the peak response, which is slightly behind the force, actually increases slightly between 20 and 40m/s. This effect may be attributed to inertia effects.

5.2 The Effect of Frequency

The transverse strain due to static, sinusoidally varying forces with amplitudes of 1kN were calculated in a similar way. These results are shown in figure 8 which also shows the response to a constant load moving at 5m/s for comparison. The peak responses at 5 and 20Hz are very similar but the peak response at 10Hz is approximately 6% smaller. Again, inertia effects may be responsible for this phenomenon.

5.3 Combined Speed and Frequency Effects

Vehicles load pavements with a combination of a static axle load and a dynamic variation about it. To investigate how this combination affects pavements calculations were carried out with loads described by:

$$P(t) = 10 + \sin(2\pi f t) \ldots kN \tag{6}$$

where f is the frequency of the dynamic component.

Figure 9 shows the transverse strain at a point on the road as a load travelling at 5m/s with a dynamic force component at 15Hz passes*. The response to a constant load is also shown. The pavement response to the dynamic component is clearly visible.

Figure 10 shows a similar response with a dynamic component at 15Hz but the speed is now 15m/s. Again the dynamic component is clearly visible in the response.

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* Note that this is an extreme scenario in practice as such high frequency dynamic loads are usually only significant at high vehicle speeds. Even then *wheel hop* frequencies are usually observed to be 5–12Hz[**12**].

Figure 11 shows the response for a speed of 15m/s and a frequency of 5Hz. The response is calculated at a point on the road where the dynamic component is close to its peak. The response to a constant load of 11kN moving at the same speed is also shown on this graph. Under these conditions of low frequency and high speed the road response may be approximated by the response to the instantaneously applied load and Eq. 2 may be simplified to the *quasi-static* calculation

$$y(\mathbf{x},t) = P(t)\int_{-\infty}^{\infty} h(\mathbf{x} - \mathbf{D} - V\tau\mathbf{i}, t-\tau)\,d\tau \tag{7}$$

which requires fewer multiplications and may therefore be evaluated more economically.

Figure 12 shows the errors in the dynamic component of road response caused by using this *quasi-static* calculation for loads at different frequencies and speeds. It is clear that some caution must be exercised before using the above equation but as the dynamic load is of the order of 10% of the static load the overall errors are expected to be small (less than 1%) for most realistic operating conditions.

6. CONCLUSIONS

Two instrumented sections of road on the TRRL test track were tested for isotropy and linearity to investigate the validity of using a convolution calculation to simulate road responses. These sections proved to be essentially linear and isotropic. They were, however, not of typical UK trunk road construction and further investigation is required before extrapolation to all roads can be guaranteed.

The theoretical road response calculation was validated by comparison with full scale tests. Good agreement was found between experiment and theory and some sources of error were discussed.

The theoretical response of a test section to constant moving loads, sinusoidal stationary loads and combinations of the two were calculated. These showed that the peak response decreased by approximately 20% between 0 and 40m/s and by approximately 16% between 0 and 20Hz. This is likely to have a substantial effect on road damage due to dynamic wheel loads.

When low frequency, fast moving loads are applied to a road the convolution calculation may be simplified and speeded up.

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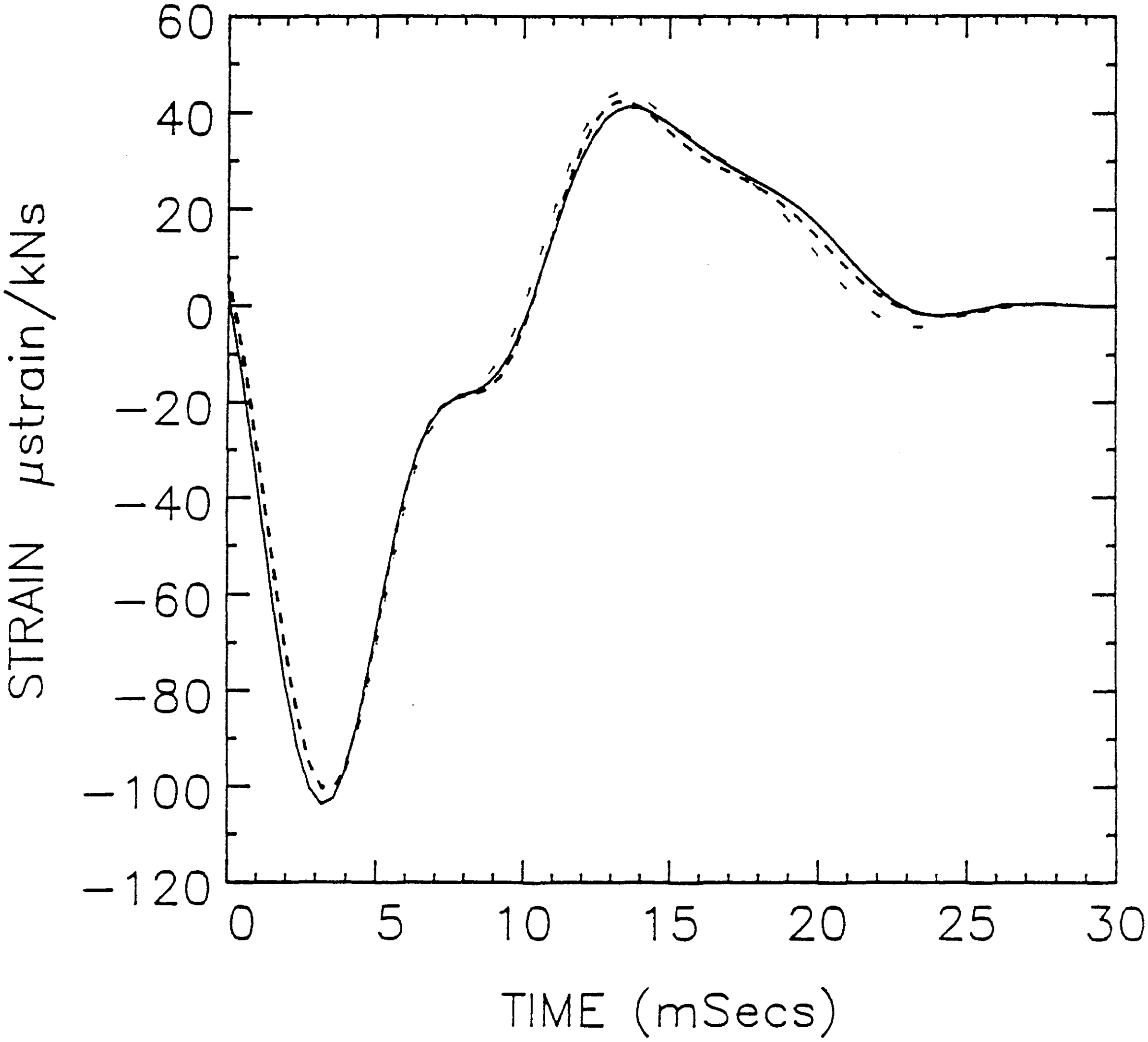

Fig. 1A. Normalised impulse responses due to different sized impulses on test section A
Hammer height: – – – – 0.5m, – – – – – – – – 1m, ———— 2m.

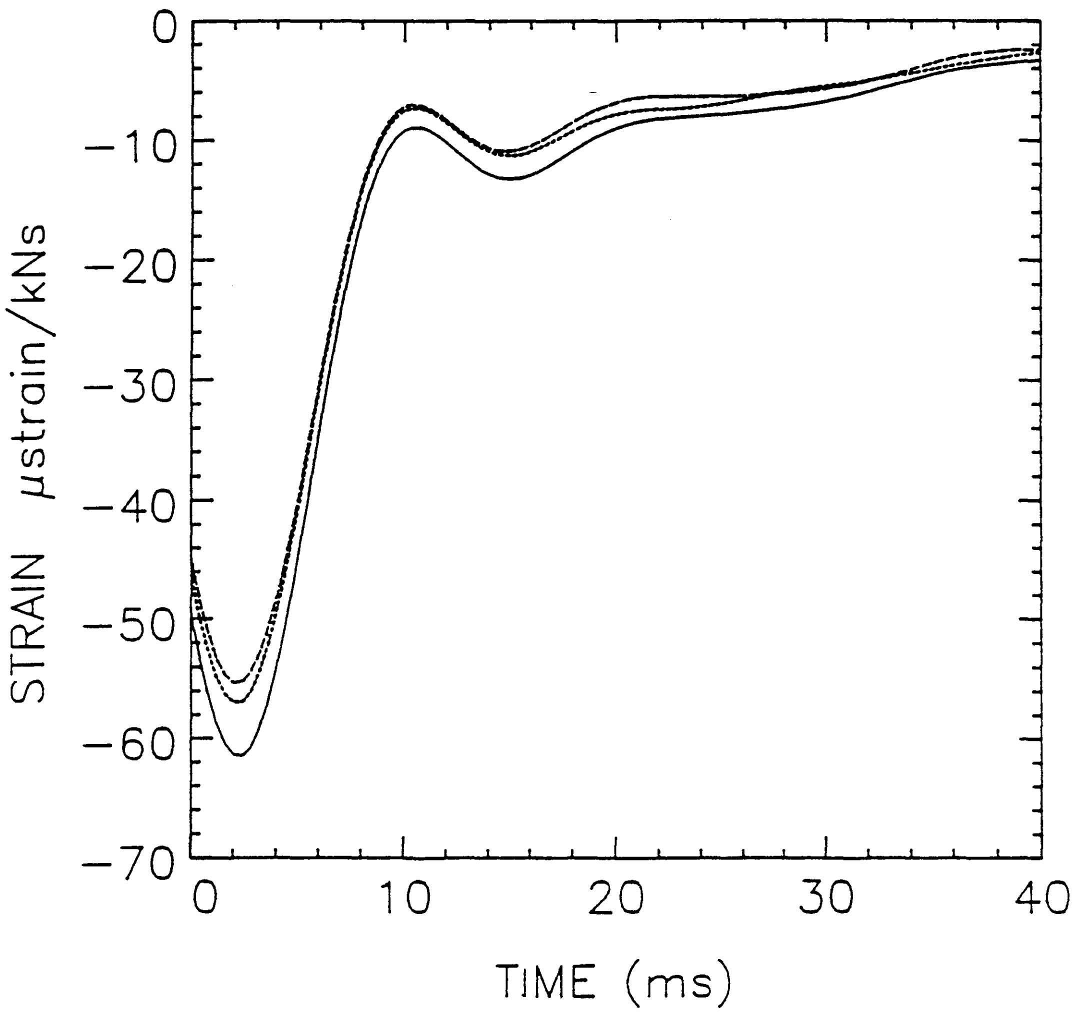

Fig. 1B. Normalised impulse responses due to different sized impulses on test section B
Hammer height: ---------0.5m, -------------1m, ————— 2m.

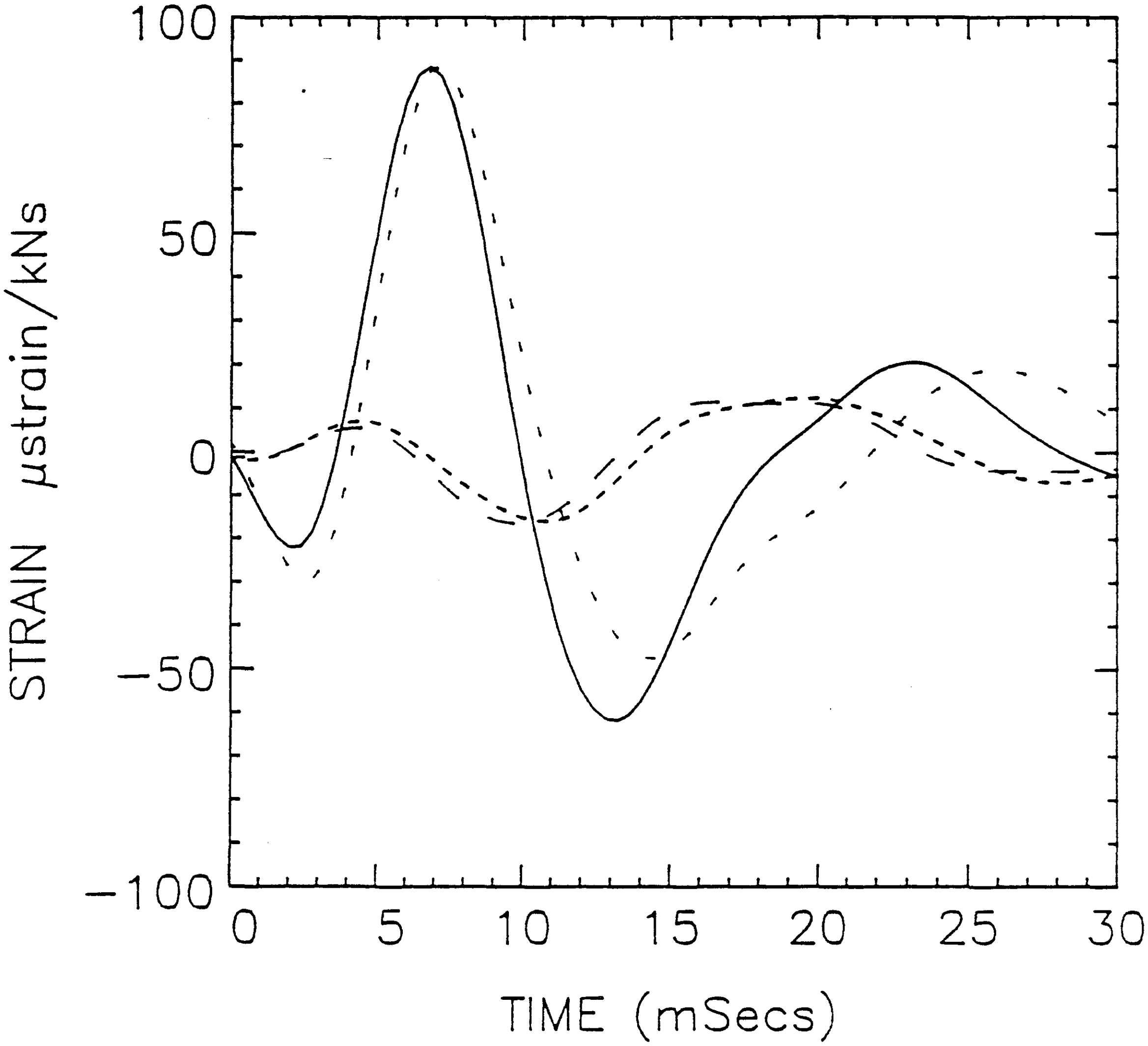

Fig. 2A. Isotropy tests at 2m on test section A
——*and*- - -*Longitudinal strains*,— — *and*- - - -*Transverse strains.*

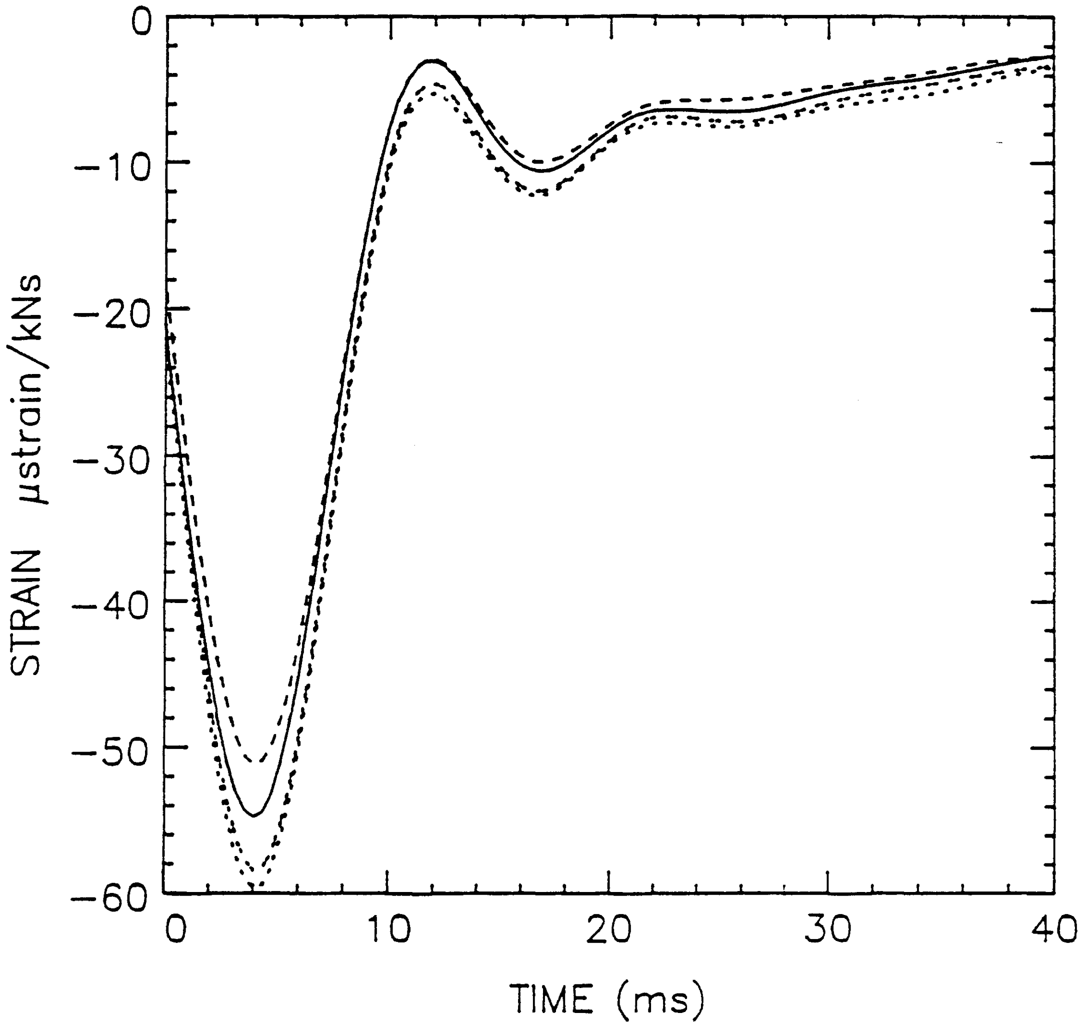

Fig. 2B. Isotropy tests at 100mm on test section B
——*and*– – – -*Longitudinal strains*,- - - - - - *and*·· ·· ·· -*Transverse strains.*

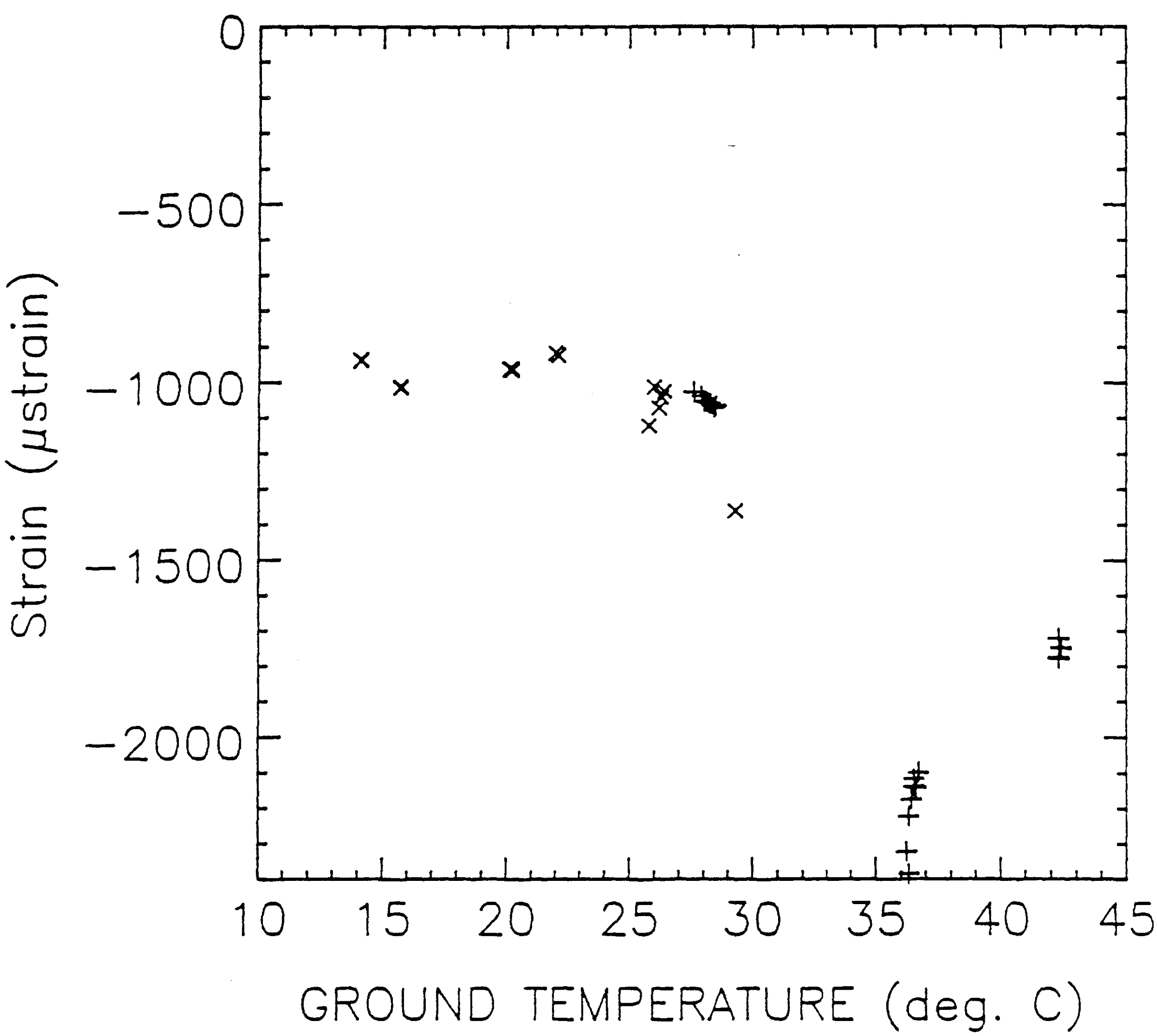

Fig. 3. Peak impulse response as a function of surface temperature for section A.

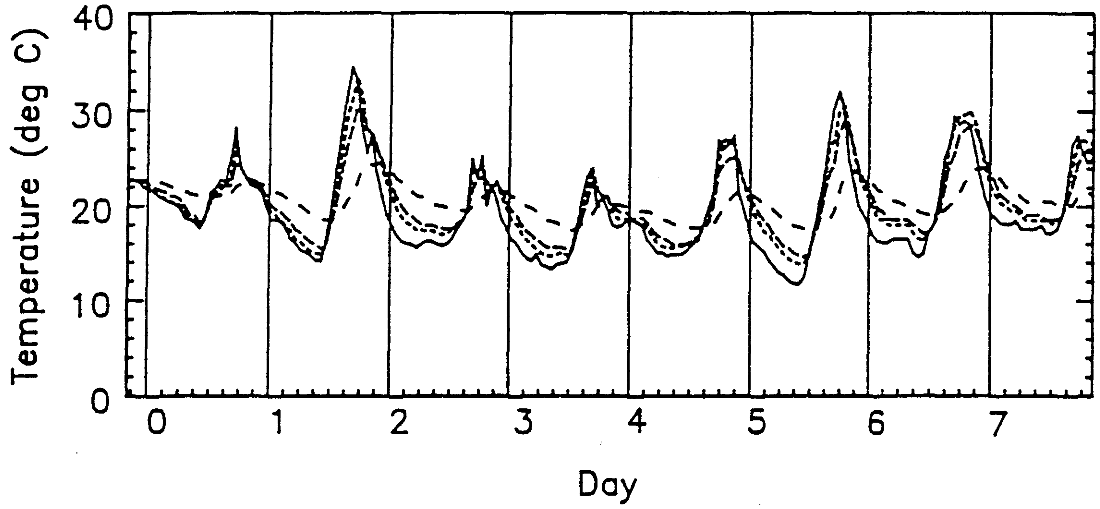

Fig. 4. Daily variation of temperature at different depths through the road structure
Depth: ——— *5cm,* --------- *12cm,* - - - - - - *20cm,* – – – *50cm.*

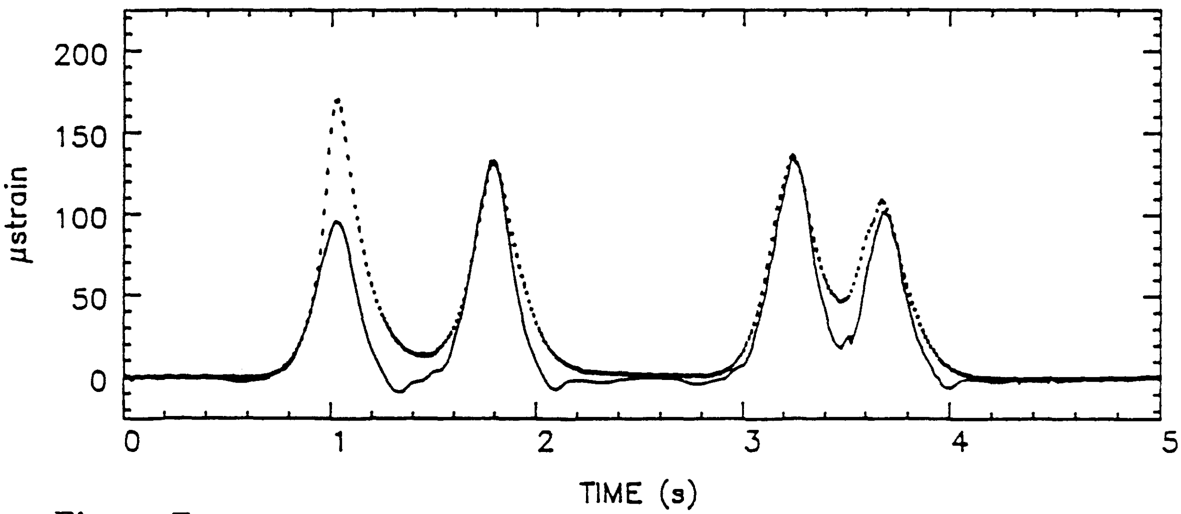

Fig. 5. Transverse strain response for a vehicle speed of 15km/h
Calculated Response ——— *Measured Response* - - - - - - - -

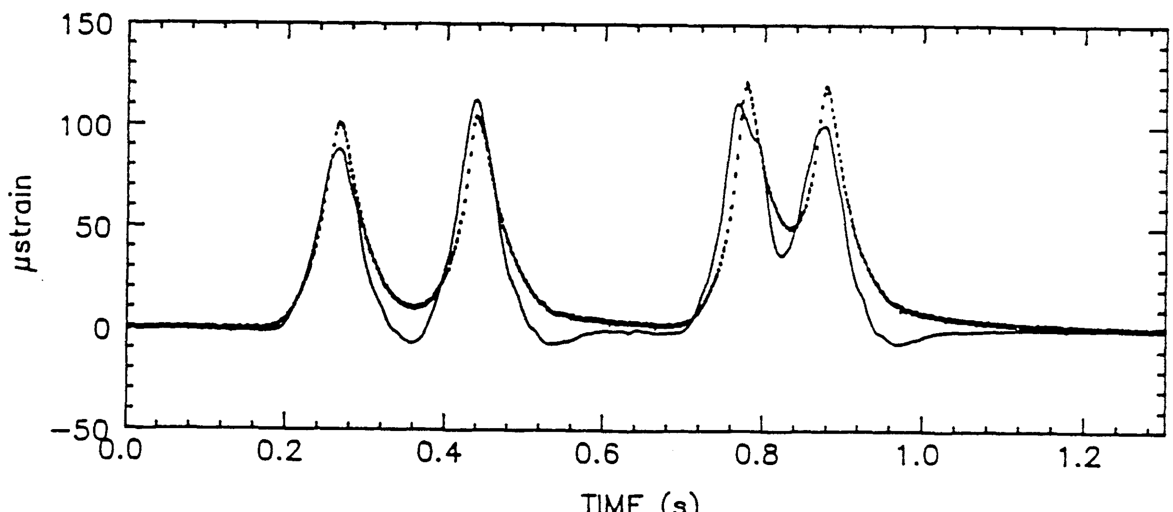

Fig. 6. Transverse strain response for a vehicle speed of 80km/h
Calculated Response ——— *Measured Response* - - - - - - - - -

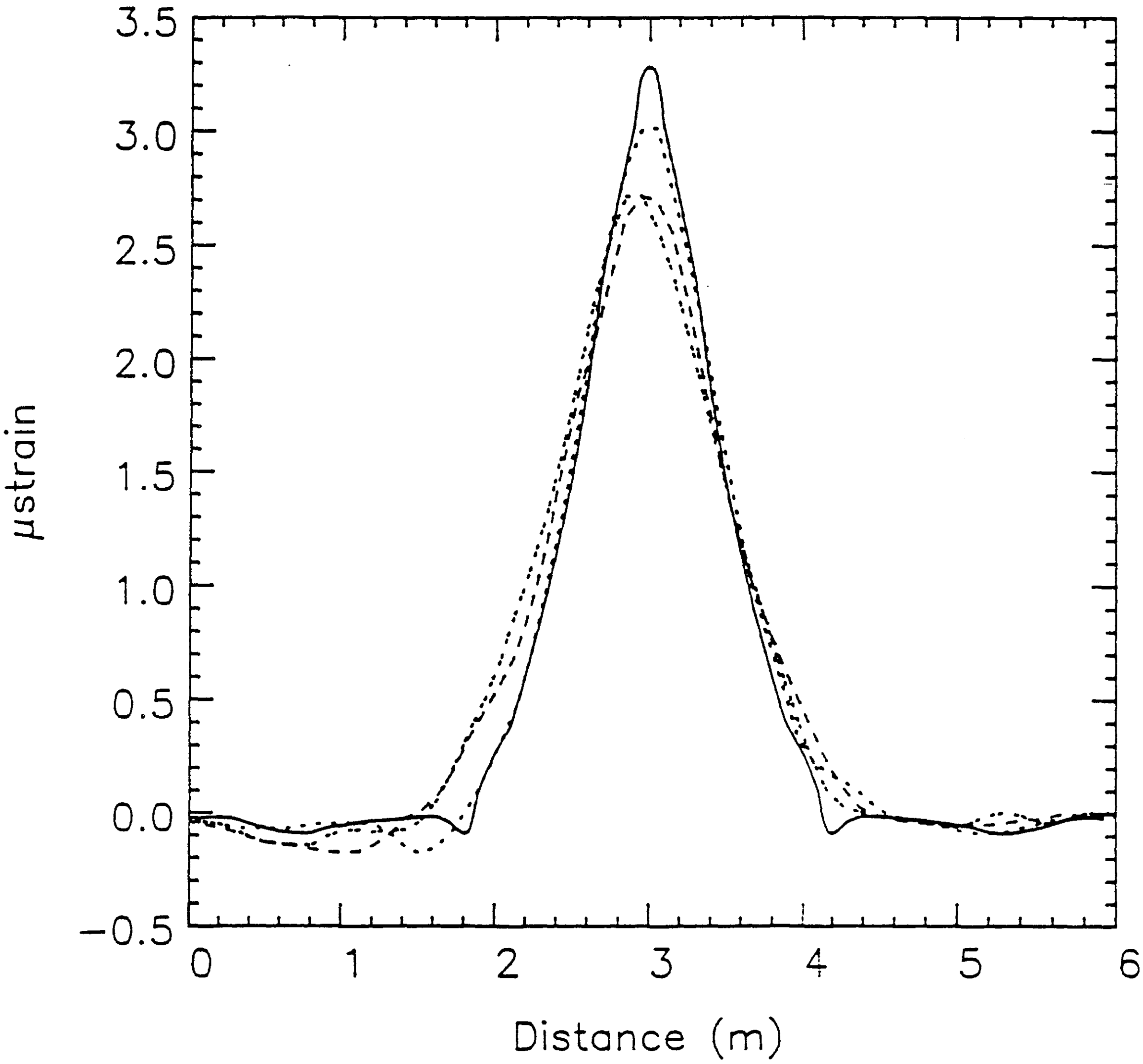

Fig. 7. The effect of speed on predicted strain response
Speed: ——— *0m/s*, - - - - - - - *5m/s*, – – – – – *20m/s*, ·· ·· ·· *40m/s*.

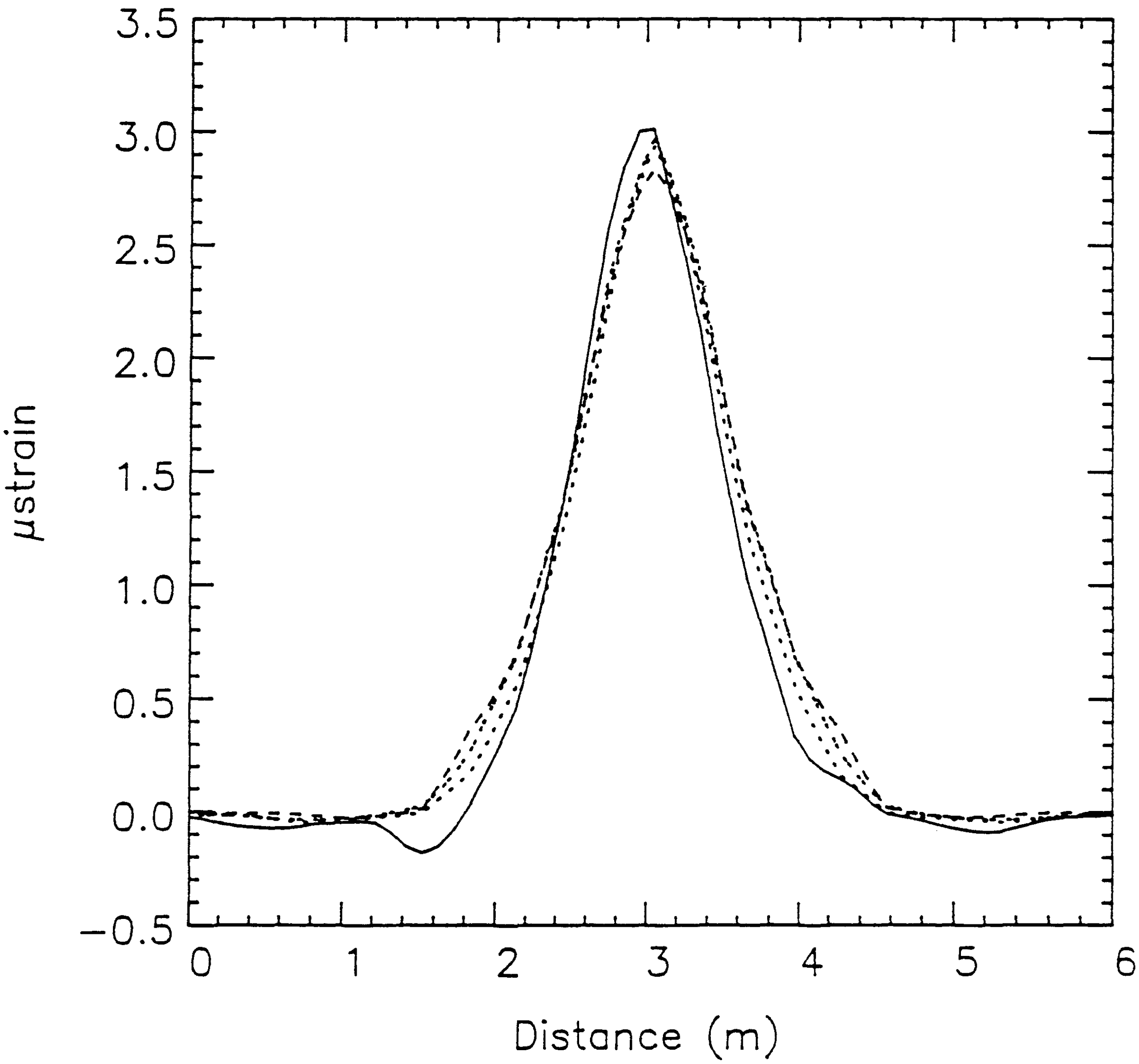

Fig. 8. The effect of frequency on predicted strain response
Frequency: ——— *5m/s, 0Hz,* - - - - - - *5Hz,* – – – – *10Hz,* ·· ·· ·· *20Hz.*

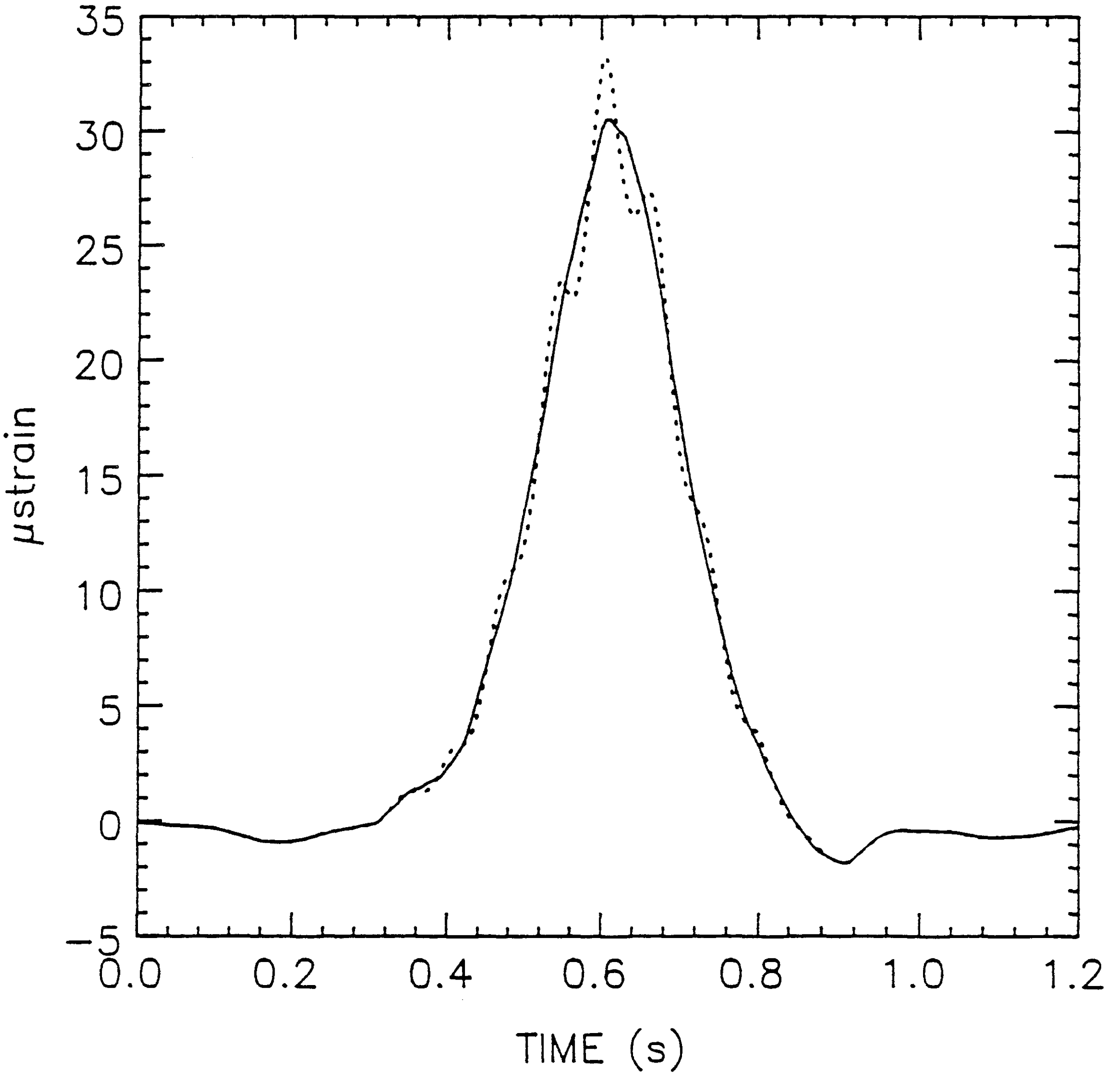

Fig. 9. The response due to a moving dynamic load, 5m/s, 15Hz
——*10kN constant load,*- - - - - -*with 10% dynamic load.*

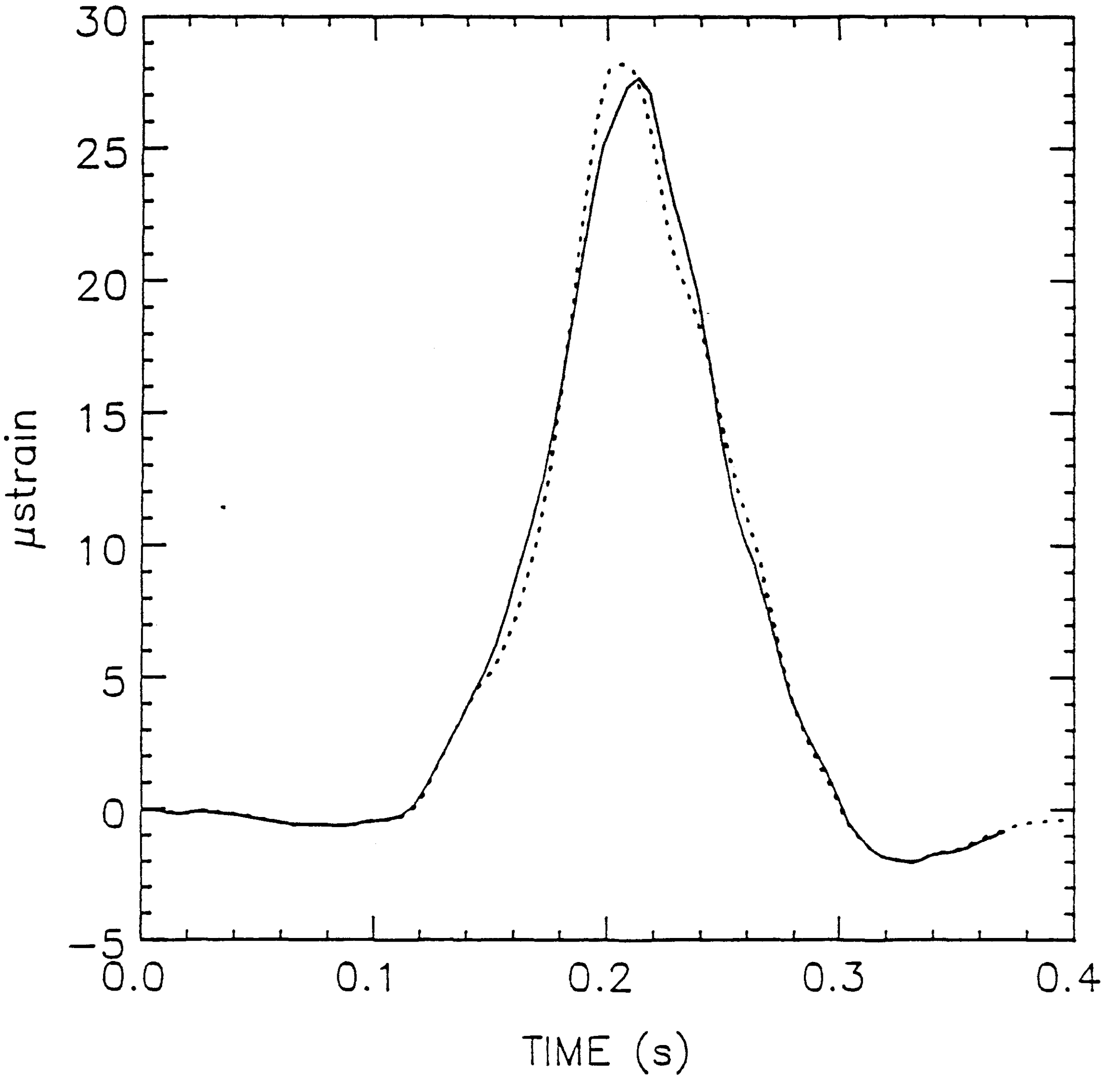

Fig. 10. The response due to a moving dynamic load, 15m/s, 15Hz
——*10kN constant load,* - - - - - - *with 10% dynamic load.*

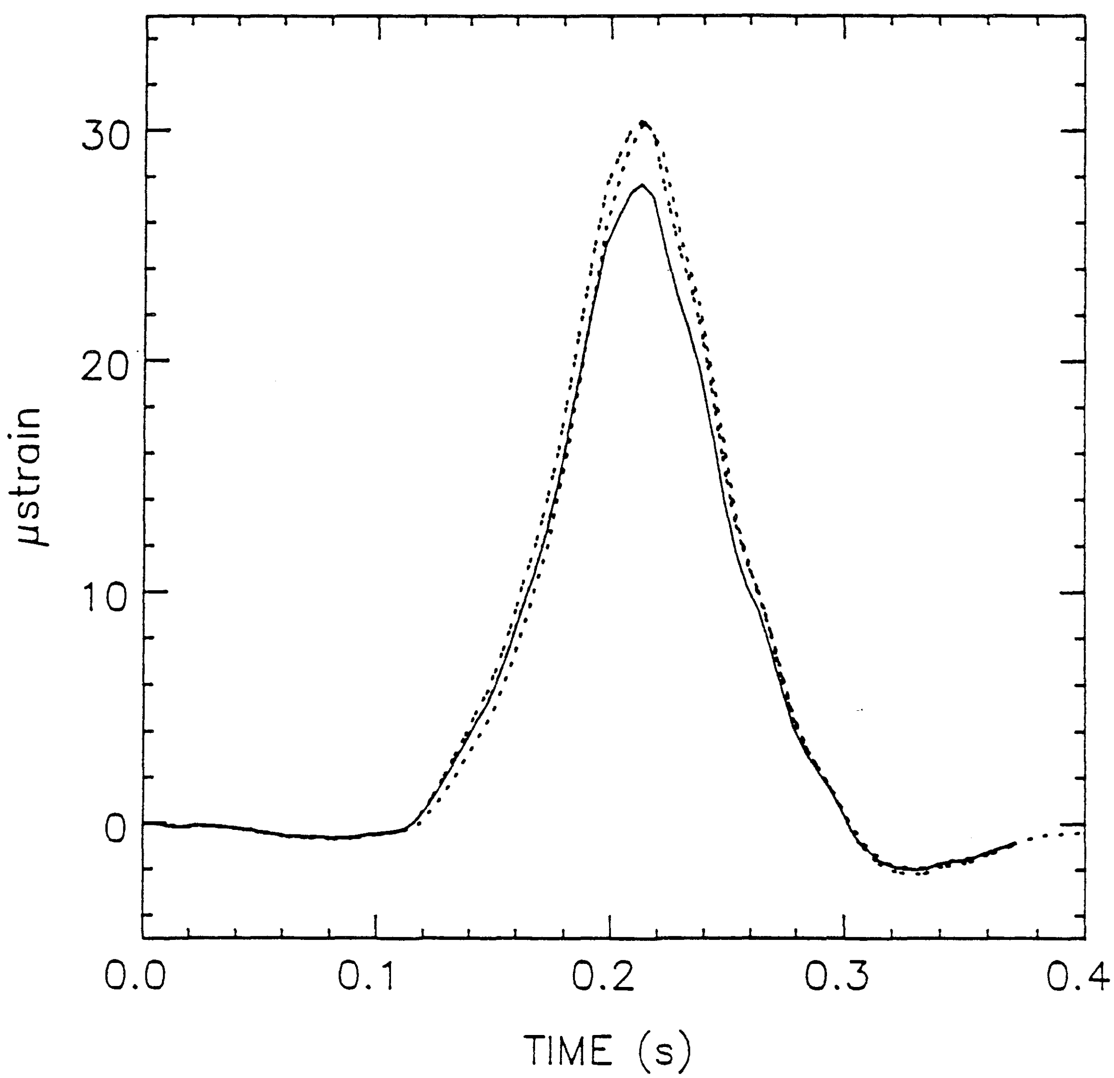

Fig. 11. The response due to a moving dynamic load, 15m/s, 5Hz
——*10kN constant load,* - - - - - - *with 10% dynamic load,*
·· ····*11kN constant load.*

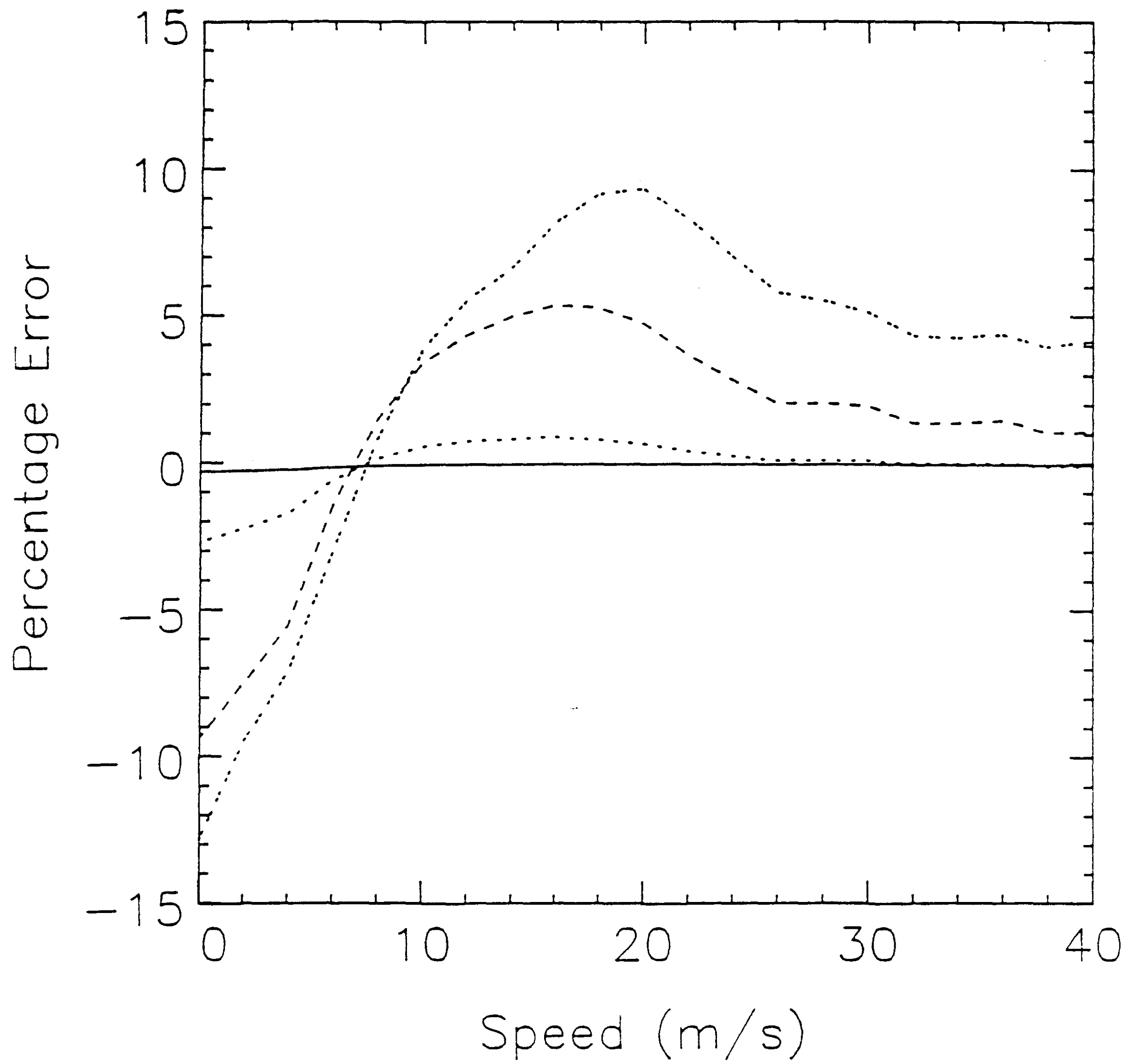

Fig. 12. The errors caused by a quasi-static calculation
Frequency:——— 0.5Hz,· · · · · · 2Hz,– – – – 5Hz,·– ·– ·– ·–10Hz.