

ON THE STABILITY OF HEAVY ARTICULATED LIQUID TANK VEHICLES

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## NOMENCLATURE

$a_y$	= Vehicle Lateral Acceleration (g)
$b_i$	= Half of the lateral distance between inner tyres on axle $i$ (m)
$d_i$	= Dual tyre spacing of tyres on axle $i$ (m)
$F_{y_i}$	= Lateral force developed at the tyre road interface (N)
$FR_i$	= Lateral forces acting through the roll centre (N)
$FS_{ij}$	= Force due to spring $j$ on axle $i$ (N)
$FT_{ij}$	= Vertical force due to tyre $j$ on axle $i$ (N)
$H_i$	= Vertical height of $i$ th unsprung mass centre from the ground plane (m)
$H_{ri}$	= Vertical height of the roll centre from the ground plane (m)
$R_{ti}$	= Effective radius of tyres on axle $i$ (m)
$S_i$	= Half the lateral suspension spread (m)
$W_\ell$	= Total weight of liquid carried (N)
$W_t$	= Sprung weight of the tank-semitrailer structure (N)
$W_{ui}$	= Total unsprung weight of axle $i$ (N)
$\bar{Y}$	= Lateral location of the mass centre of liquid (m)
$\bar{Z}$	= Vertical location of the mass centre of liquid (m)
$\bar{Z}_0$	= Vertical location of the mass centre of liquid, for $\theta_{s3} = a_y = 0$ (m)
$Z_{ri}$	= Roll centre height (m)
$Z_{ui}$	= Vertical location of the unsprung mass centre of gravity (m)
$\theta_{si}$	= Roll angle of sprung weight $i$ (rad)
$\theta_{ui}$	= Roll angle of unsprung weight $i$ (rad)
$\phi$	= Inclination of the free surface (rad)

## ABSTRACT

The dynamics associated with the bulk transportation of liquid is unique in that the slosh forces associated with the motion of the liquid within the tank vehicle significantly affects the controllability/manoeuvrability of the tank vehicles. The roll and lateral stability of an articulated tank vehicle with partial liquid load is discussed from the view point of fundamental mechanics of vehicle response and quantitative influence of size and weight variables. The influence of the liquid motion within the tank, on the steady turning (kineto-static) stability of the vehicle is investigated, assuming inviscid fluid flow conditions. A roll plane model of a partially filled tank is developed and integrated with the static roll plane model of the tank vehicle to study the influence of the liquid slosh forces. The study reveals that the rollover threshold acceleration of the partially filled liquid tank vehicle is considerably affected by the motion of the free surface of liquid within the tank. The magnitude of liquid load shift during a steady turning manoeuvre depends on the tank cross-section, fill condition, tank roll and the lateral acceleration imposed on the liquid. The rollover threshold acceleration levels of the liquid tank vehicle is compared with that of an

equivalent rigid cargo vehicle. The influence of tank geometry, liquid fill level and the compartmenting of the tank on the rollover threshold acceleration is also investigated. The study of the dynamic sloshing effects of the liquid within a moving vehicle is investigated by integrating a quasi-dynamic fluid model of the tank and a three-dimensional dynamic vehicle model. The dynamic response parameters of the tank vehicle for typical highway manoeuvres are obtained and compared to that of an equivalent rigid cargo vehicle in order to study the influence of the destabilizing effects of the liquid load shift. The dynamic response characteristics of the tank vehicle depends on the vehicle speed, tank fill and the steer input. A comprehensive discussion on the relative performance of the liquid tank vehicle and the rigid cargo vehicle is presented with significant highlights.

## INTRODUCTION

The recent changes in the allowable weights and dimensions of the heavy freight vehicles has prompted a strong interest to the trucking industries to increase the capacity of the vehicles in order to economize the freight transportation. Increase in the size and weight of heavy vehicles, specifically the tank vehicles, result in significant changes in the control and manoeuvrability of the heavy vehicles. The rollover immunity levels of heavy vehicles, in general, are found to be so low that a moderately severe manoeuvre could lead to instability and rollover [1]\*. Hence, safe transportation of bulk liquids in highway tank vehicle deals with factors other than normal trucking practices. A vast majority of studies on the dynamics of heavy vehicles focus on the stability and handling of vehicles carrying rigid cargo [2,3,4]. Rigid cargo, in general, is fastened to the trailer and hence the cargo interaction with the vehicle is considerably negligible. However, in case of liquid tank vehicles, the dynamic interactions arising due to the motion of liquid within the tank cannot be ignored. The liquid motion within the moving tank vehicle may cause excessive yaw, roll and lateral motions of the vehicle. Certain vehicle manoeuvres at typical highway speeds coupled with the liquid slosh can lead to increased forces and moments on the vehicle leading to vehicle instability and rollover.

The study of sloshing effects of the liquid on the stability of tank vehicles involves highly complex modelling and analyses. The sloshing effects within the tanks has been characterized through equivalent mechanical models derived from the fluid dynamics equations [5,6]. While a comprehensive model representing the interactions of liquid on the vehicle dynamic response may be of highly complex nature, the vehicle response parameters can be established by interfacing steady state fluid motion to the vehicle model. In this paper, two models of the tank vehicle are presented, one to establish the rollover threshold acceleration during steady turning and the other, investigates the dynamic response of the tank vehicles during various highway manoeuvres.

## KINETO-STATIC ROLL PLANE ANALYSIS OF ARTICULATED TANK VEHICLES

A computer simulation model of an articulated vehicle with partially filled arbitrarily shaped tank is developed to investigate the steady turning behaviour of the vehicle, assuming inviscid fluid flow conditions. During a steady turning process, the tank vehicle is subjected to a roll and lateral acceleration. The liquid within the tank experiences a shift due to the roll of the sprung mass, and the acceleration imposed on the liquid shifts the liquid bulk further, as shown in Figure 1. The gradient of free surface of liquid with reference to the tank axes is determined from the following

\* Numbers in square brackets indicate references at the end of the paper.

equation [7]:

$$\phi = \frac{a_y + \theta_{s3}}{1 - a_y \theta_{s3}} \quad (1)$$

where  $a_y$  is the lateral acceleration imposed on the liquid mass,  $\theta_{s3}$  is the tank roll angle and  $\phi$  is the free surface gradient. Assuming the entire fluid bulk to move as a rigid body, the load shift associated with the steady turning process is computed and integrated into the roll plane model of the vehicle to investigate the rollover threshold limits of the tank vehicle. The associated lateral and vertical translations of the centre of mass of liquid bulk depend on the fill level, tank cross-section and the free surface gradient of the liquid  $\phi$ . The computation of the centre of mass of liquid within a circular cross-section tank can be computed from a set of closed form equations [7]. However, in case of tanks of arbitrary cross-section, the computation of the instantaneous centre of mass of liquid yields significant complexities [8]. A numerical iterative technique is formulated to compute the centre of mass of liquid within tanks of arbitrary cross-section. The roll plane model of the partially filled tank of arbitrary cross-section is then integrated with the static roll model of the tank vehicle to evaluate the rollover threshold acceleration.

Tank vehicles with multiple axles are often approximated by grouping the axles such that the multiple axles are reduced to three composite axles. The suspension properties of the composite axles are obtained by combining the properties of the individual axles, grouped to form the composite axle. The composite axles represent the tractor front, tractor rear and the trailer axles. The tractor sprung weight is modelled as two sprung weights supported at the tractor front and rear axles and the two weights are coupled through the torsional flexibility of the tractor frame. The tank-trailer sprung weight is modelled as  $(n+1)$  sprung weights, representing the tank-trailer tare sprung weight and the liquid loads in the  $n$  compartments. The trailer sprung weight is coupled to the tractor rear sprung weight through a torsional spring representing the fifth wheel and tank-trailer structural compliance. Figure 2 shows the forces and moments acting on the axle  $i$  in the roll plane of the tank-trailer. The equations of motion of the tank vehicle is defined by balancing the roll moments acting on the sprung and unsprung weights, the vertical forces acting on the suspensions and tires, and the lateral forces acting on the tires. The complete derivation of the equations of motion is presented in the reference [7]. The 15 equilibrium equations along with the  $2n$  equations necessary to obtain the vertical and lateral shift of centre of mass of liquid in the various compartments of the tank is written in the matrix form as:

$$[A]\{\Delta x\} = \{B\} \Delta \theta_{s3} \quad (2)$$

where the matrix  $[A]$  is a  $[(15+2n) \times (15+2n)]$  coefficient matrix and  $\{B\}$  is a  $(15+2n)$  vector of vehicle weights and dimensions.  $\{\Delta x\}$  defines the change in the vehicle responses such as the roll angles of the sprung weights and unsprung weights, translations of the roll centre with respect to the unsprung mass, tire vertical and lateral displacements and the instantaneous centre of mass of liquid within the compartments, caused by varying the roll angle of the tank-trailer roll angle  $\Delta \theta_{s3}$ . Equation (2) is solved to compute the vehicle response parameters due to change in the roll angle of the tank-trailer. The response parameters and the coefficients of the matrix  $[A]$

and the vector {B} are updated for every increment of the roll angle. The calculations are terminated when the trailer composite axle and the tractor rear composite axle tries lift off the ground. The highest lateral acceleration encountered during the computing process determines the rollover threshold acceleration of the tank vehicle.

### DYNAMIC RESPONSE MODEL OF THE ARTICULATED TANK VEHICLE

A 5-axle tractor-semitrailer vehicle with a cleanbore circular cross-section tank is modelled to study the influence of liquid load shift on the directional dynamics of the tank vehicle. The tank vehicle model is formulated by integrating the roll plane model of the partially filled tank with the three-dimensional model of the articulated vehicle, assuming constant vehicle speed. The gradient of the free surface and the instantaneous centre of mass of liquid in the cleanbore tank is computed using the instantaneous acceleration imposed on the liquid mass and the tank roll using equation (1). The moment of inertias of the liquid cargo about its centre of gravity is obtained by splitting the liquid mass within the tank into a number of thin sections and performing the necessary transformation to the overall centre of gravity of the liquid mass. In the case of tanks with circular cross-section, due to the symmetricity in the tank shape, the moment of inertias of the deflected shape remain the same about the axis of symmetry. The moment of inertias of the deflected shape are then computed about the new rotated coordinate system by transformation of the moments of inertias about the fixed axes system using the gradient of the free surface of the fluid bulk.

The tractor-semitrailer is modelled as two sprung masses: the sprung mass of the tractor and the tare mass of the semitrailer and the empty tank. The liquid mass is modelled as a floating mass with respect to the semitrailer axes, while the axles are modeled as independent unsprung masses. The sprung and the unsprung weights are treated as rigid bodies with five (lateral, vertical, yaw, roll and pitch) and two (roll and bounce) degrees of freedom respectively. The fifth wheel constraint forces are evaluated from the kinematic expressions, relating the acceleration of the sprung weights. The constraint moments are evaluated as a function of the relative angular displacement of the tractor and the semitrailer. The equations of motion of the tank vehicle comprises of five second order differential equations for each of the two sprung masses and two equations for each of the unsprung masses, the detailed derivation of them are presented in the reference (9). Based on the steer input to the vehicle, the lateral forces and the aligning moment developed at the tire-road interface are computed along with the spring forces and constraint forces and moments. The second order differential equations of motion of the tank vehicle are then solved to compute the impact of liquid load shift on the directional response of the tank vehicle using numerical integration.

### RESULTS AND DISCUSSION

#### Rollover Threshold Acceleration Limits of Liquid Tank Vehicles:

The  $(15+2n)$  equations, describing the static roll equilibrium of a five-axle tractor-tank-semitrailer vehicle are solved simultaneously to evaluate its rollover threshold acceleration. Computer simulation are carried out for tanks of circular, modified oval, modified square and elliptical

cross-sections. The overturning lateral acceleration limit of the tank-vehicle is evaluated for various fill levels, where fill level is defined as the ratio of the liquid height to the total height of the tank. The rollover threshold acceleration limits of tank vehicles, equipped with the various tank cross-section are compared to those of a vehicle carrying an equivalent rigid cargo vehicle. Figure 3 compares the rollover threshold limits of a partially filled modified oval tank vehicle and equivalent rigid cargo vehicle for various fill condition. The rollover threshold acceleration limits of the 100% filled and empty tank vehicle and the equivalent rigid cargo vehicle are found to be the same due to the absence of the load shift. However, the rollover threshold limits of partially filled tank vehicle are significantly low compared to those of the vehicle carrying the rigid cargo, as shown in Figure 3. The reduced overturning limits of a partially filled tank vehicle is attributed to the liquid load shift occurring during the steady turning process leading to increased roll moments and cornering forces at the tire-road interface. The rollover threshold acceleration limits of the tank vehicle also depends on the cross-section of the tank. The fluid within modified oval and modified square tanks experiences larger load shift compared to circular and elliptical tank cross-sections and thus yields considerable deterioration in the steady turning process of the tank vehicle [8].

The reduced rollover threshold acceleration limits reduces the limiting speed of the tank vehicle compared to the rigid cargo vehicle while negotiating a curve of given radius. A comparison of the the limiting speeds of the tank vehicle to that of the equivalent rigid cargo vehicle for various fill conditions is presented in Figure 4. A 40% filled modified oval tank vehicle has its limiting speed reduced by approximately 15 km/h while negotiating a curve of 50 m (164 feet) radius compared to an equivalent rigid cargo vehicle.

The tank vehicles are often partially filled due to change in the weight density of the fluid carried. However, the load carried by the composite axles is maintained around a permissible value governed by the local laws. Figure 5 presents a comparison of the rollover threshold acceleration limits of the tank vehicle and the equivalent rigid cargo vehicle while the composite axle loads are held constant by varying the weight density of the liquid carried. The 100% fill condition and the corresponding composite axle loads are based upon fuel oil of weight density  $0.0068 \text{ N/cm}^3$ . The rollover threshold acceleration of the equivalent rigid cargo vehicle rapidly increases with the decrease in the fill condition due to the reduced overall centre of gravity height. However, the tank vehicle equipped with modified oval tank exhibit a gradual decrease in the rollover threshold value as the density of the liquid is increased, as shown in Figure 5. This is attributed to the fact that with the decrease in the fill condition, the liquid load shift encountered within the moving tank increases leading to increased roll moments. A comparison of the rollover immunity levels of the tank vehicles carrying various liquids, while the payload and the composite axle loads are held constant is presented in Figure 6. The circular tank vehicle shows a very little variation in the rollover threshold values when the density of the liquid is varied, while in case of modified oval tank vehicle the deviation in the threshold values is quite significant. In case of lighter liquids, the circular tank vehicle reveal the lowest rollover threshold acceleration and the modified oval tank vehicle has the best rollover threshold value. However, for denser liquids, the threshold levels are best with tank vehicles equipped with circular cross-section while the modified tank vehicle has the lowest

threshold value.

### Dynamic Response of Tank Vehicles Subject to Highway Manoeuvres:

The directional response of a five-axle, tractor-tank-semitrailer, equipped with a partially filled cylindrical cleanbore tank, is carried out via computer simulation for typical highway manoeuvres. The simulation is carried out for steady steer input, lane change and evasive manoeuvre. The vehicle response parameters such as the roll angle and lateral acceleration are obtained for both liquid cargo and the equivalent rigid cargo vehicles, to demonstrate the influence of liquid load shift on the dynamic response of the tank vehicle. The open loop steer input to the vehicle is directly applied to the front wheel of the vehicle. The directional response of the semitrailer of the 5-axle tank vehicle equipped with partially filled circular tank subjected to the steady steer inputs is presented in Figure 7. The roll angle and the lateral acceleration response of the 40% and 70% filled tank-semitrailers do not deviate significantly from that of the equivalent rigid cargo vehicle for 1° steer input. However, the roll angle as well as the lateral acceleration response of the partially filled tank-semitrailer is considerably larger than that of the rigid cargo vehicle, when the steer is increased to 4°. The effect of the liquid load shift is more apparent for a 70% filled semitrailer; the liquid tank vehicle becomes unstable and tends to rollover. The equivalent rigid cargo vehicle also encounters increased roll and lateral acceleration and a lift-off of the rearmost trailer axle.

The load shift occurring during a steady turning process increases the cornering forces on the outer track of the vehicle leading to considerable deviation in the path followed by the liquid tank vehicle. A comparison of the path followed by the partially filled tank vehicle and the equivalent rigid cargo vehicle subjected to a steady steer input of 4° is presented in Figure 8. Figure 8 reveals that 40% filled tank vehicle deviates considerably from the path followed by the equivalent rigid cargo vehicle. The tank vehicle exhibits a deviation of approximately 10 meters in the lateral position compared to that of the equivalent rigid cargo vehicle. The excessive load shift and roll moments in case of the 70% filled tank vehicle leads to vehicle rollover, while the rigid cargo vehicle is still stable, as shown in Figure 8.

The roll angle and the lateral acceleration response of the partially filled 5-axle tank vehicle for a typical lane change manoeuvre is compared to that of an equivalent rigid cargo vehicle, as shown in Figure 9. The dynamic steer input to the vehicle to follow the specified path is computed using a "Driver model". The roll angle and the lateral acceleration response of the 40% filled and 70% filled tank vehicles are compared to that of the equivalent rigid cargo vehicle. The peak roll angle response of the liquid tank vehicles are higher compared to that of the rigid cargo vehicles, by approximately 0.4 degrees at a vehicle speed of 90 km/h. The lateral acceleration response of the tank vehicle shows a very small increase compared to that of the rigid cargo vehicle. The tank vehicles are found to be relatively stable during the lane change manoeuvre. However the change in the vehicle speed will adversely affect the dynamic response of the tank vehicle.

The evasive manoeuvre or a double lane change manoeuvre is an emergency steering manoeuvre required to avoid an obstacle in the lane of travel and



return to that lane immediately afterwards. Simulations are carried out on the tank vehicle under varying fill conditions, at a vehicle speed of 90 km/h. A comparison of the roll angle and lateral acceleration response of the tank vehicle are obtained and compared to that of the rigid cargo vehicle, as shown in Figure 10. The excessive load and associated roll moments due to the liquid load shift increases the roll angle response of the tank vehicle. In case of the 40% filled tank vehicle, the increase in the peak roll response is approximately 2 degrees while in case of the 70% filled tank vehicle, the vehicle goes unstable and rolls over, as shown in Figure 10. The lateral acceleration response of the 70% filled rigid and liquid cargo trailers are higher compared to that of the 40% filled trailers, also the 70% filled tank vehicle experiences wheel lift off leading to roll over after 7 seconds.

### CONCLUSIONS

The destabilizing forces and moments arising due to the liquid load shift within a partially filled tank adversely affect the handling and control of the liquid tank vehicle. The magnitude of liquid load shift within the tank is a function of the tank cross-section, fill level, roll angle of the trailer unit and the lateral acceleration imposed on the liquid. A kineto-static roll plane model of the tank vehicle is developed by integrating the roll plane model of the tank to that of the static roll plane model of the vehicle. The motion of the free surface of liquid within the moving tank lowers the rollover threshold acceleration limits of the liquid tank vehicle compared to that of the equivalent rigid cargo vehicle. The limiting speed of the tank vehicle in order to negotiate a curve of given radius is computed and compared to that of the rigid cargo vehicle. The influence of the tank cross-section on the rollover threshold limits is also investigated. The study of the dynamic sloshing effects of the liquid cargo on the vehicle dynamics is investigated using a quasi-dynamic model, developed by integrating a quasi-dynamic fluid model with the three-dimensional model of the tank vehicle. The dynamic response parameters, such as the roll angle and lateral acceleration response of the tank vehicle is computed and compared to that of the equivalent rigid cargo vehicle in order to investigate the influence of the liquid slosh forces. The increased slosh forces and roll moments arising within the partially filled tank increases the roll and lateral acceleration response of the vehicle leading to instability and possible rollover during emergency type of manoeuvres. A comprehensive study on the tank vehicle performance during various highway manoeuvres is presented in order to study the effects of the liquid slosh forces.

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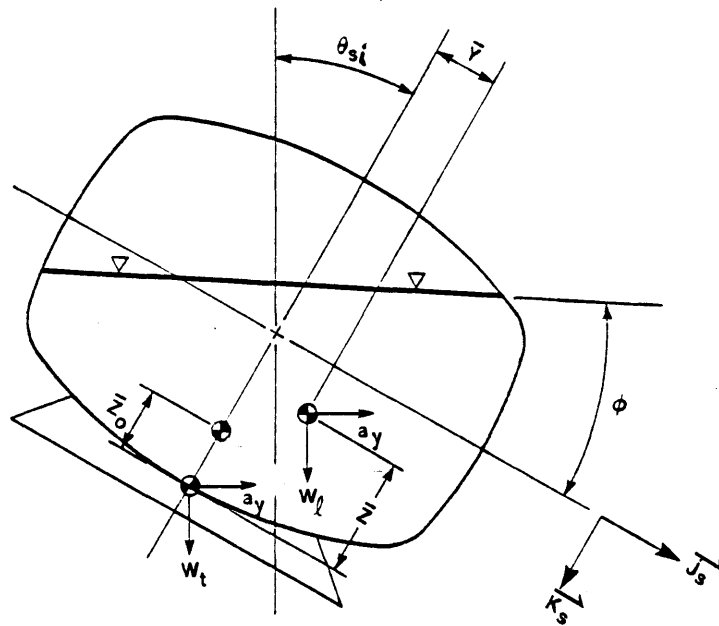


Fig. 1 Roll Plane Model of a Partially Filled Arbitrarily Shaped Tank.

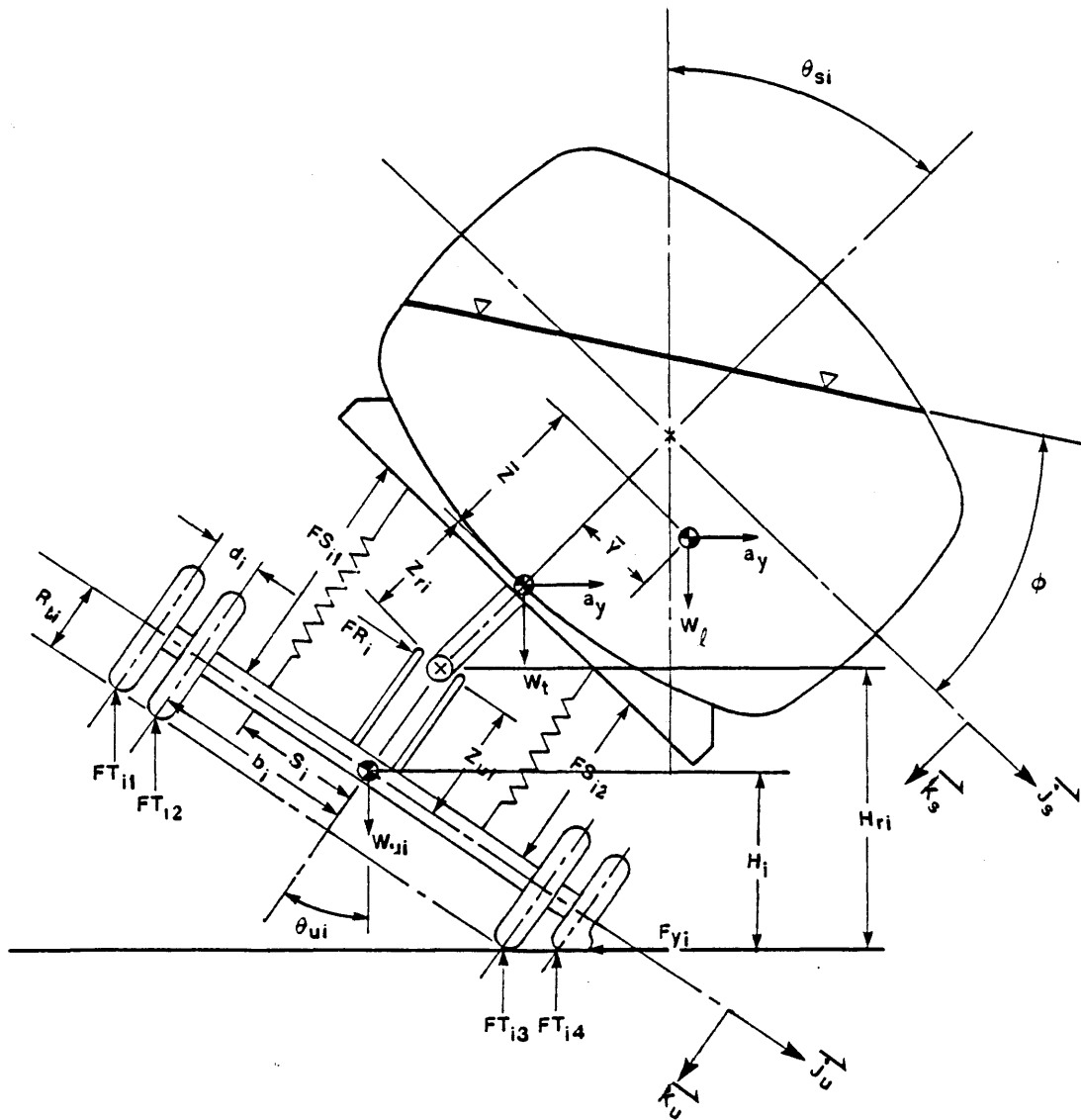


Fig.2 Forces and Moments acting on axle 'i' in the Roll Plane of Tank Vehicle

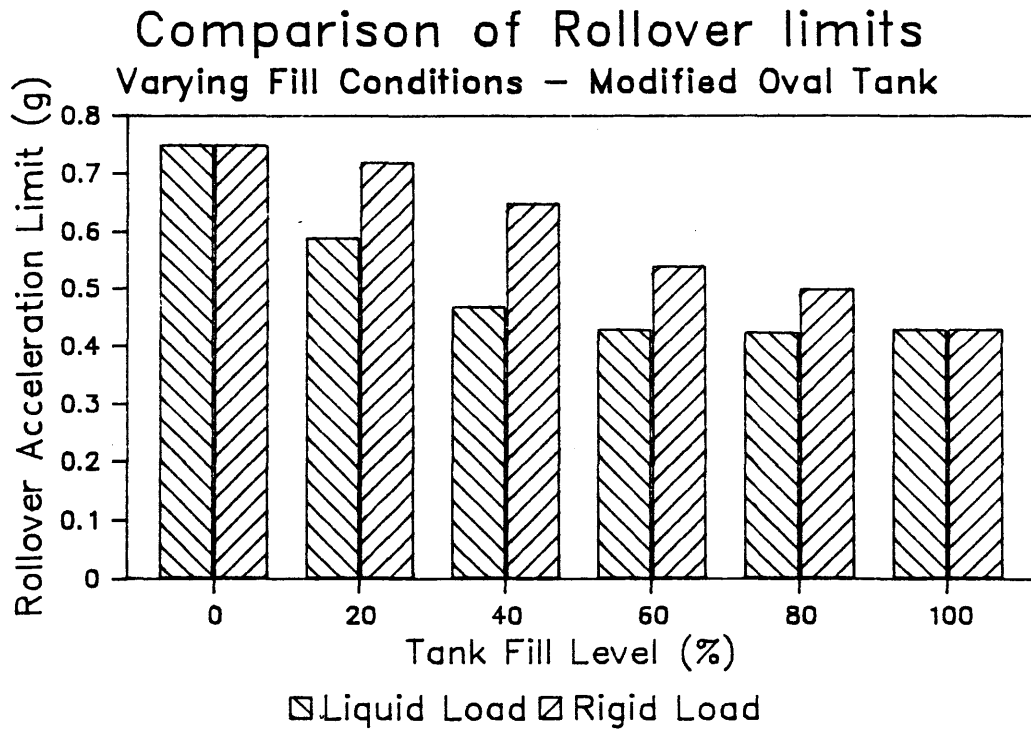


Fig. 3 Comparison of Rollover Limits of Tank Vehicle and Equivalent Rigid Cargo Vehicle: Varying Fill Conditions.

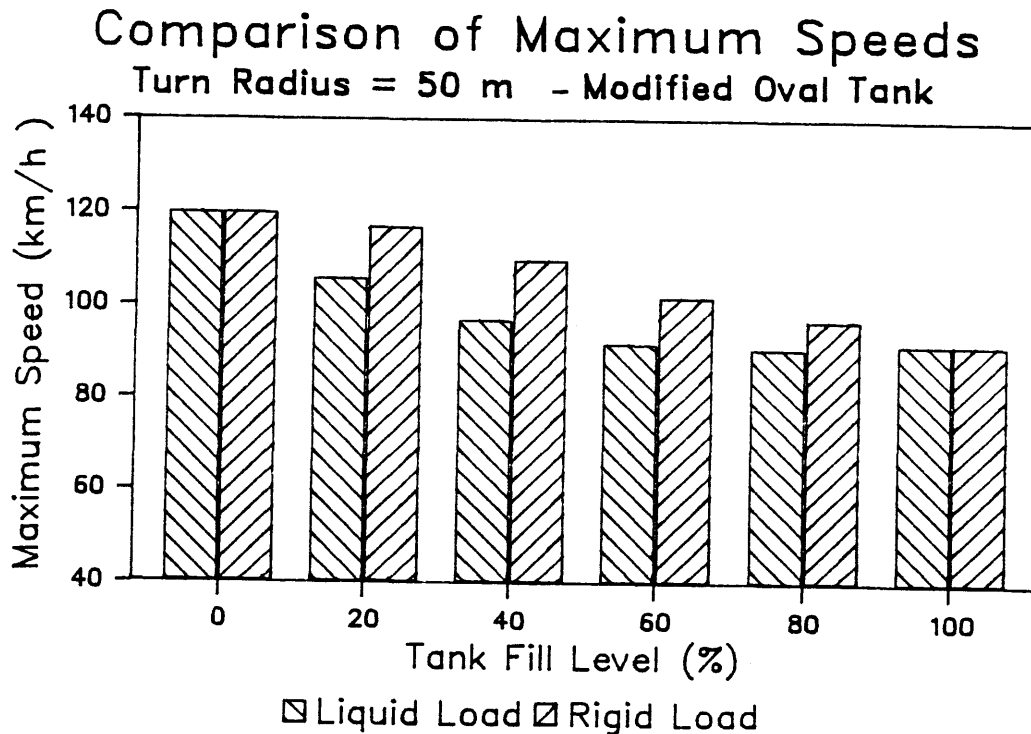


Fig. 4 Comparison of Limiting Speeds of Tank Vehicle and Equivalent Rigid Cargo Vehicle: Varying Fill Conditions.

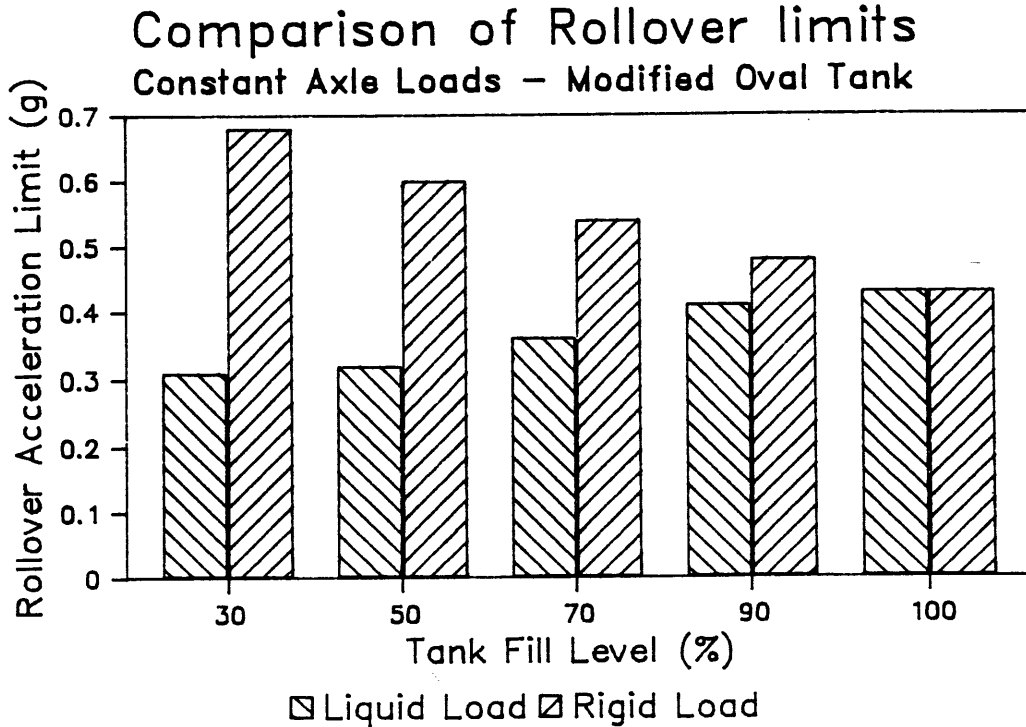


Fig. 5 Comparison of Rollover Acceleration Limits of Tank Vehicle and Equivalent Rigid Cargo Vehicle : Constant Axle Loads.

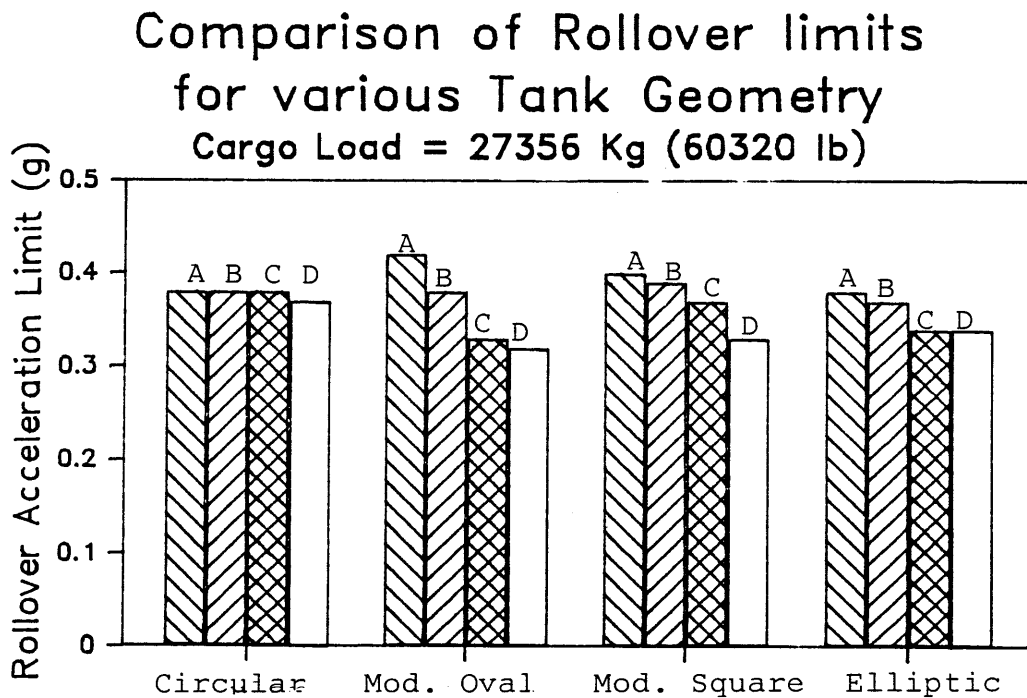


Fig. 6 Comparison of Rollover Limits of Tank Vehicles with Various Tank Cross-sections carrying different Liquids.

A - Fuel Oil : B - Domestic Oil : C - Diesel : D - Industrial Acid

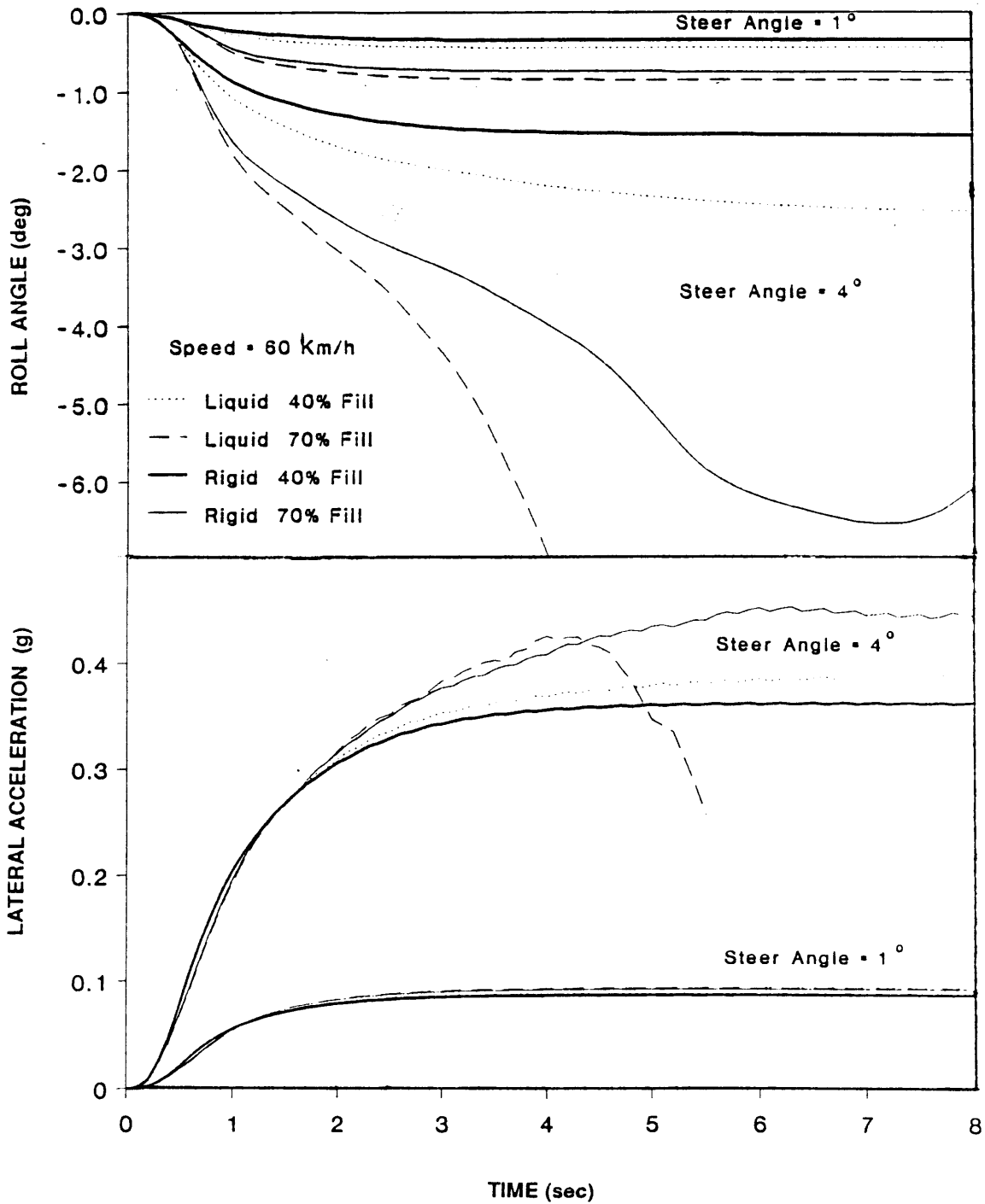


Fig. 7 Comparison of Dynamic Response of Tank Vehicle and Equivalent Rigid Cargo Vehicle subjected to Steady Steer Inputs.

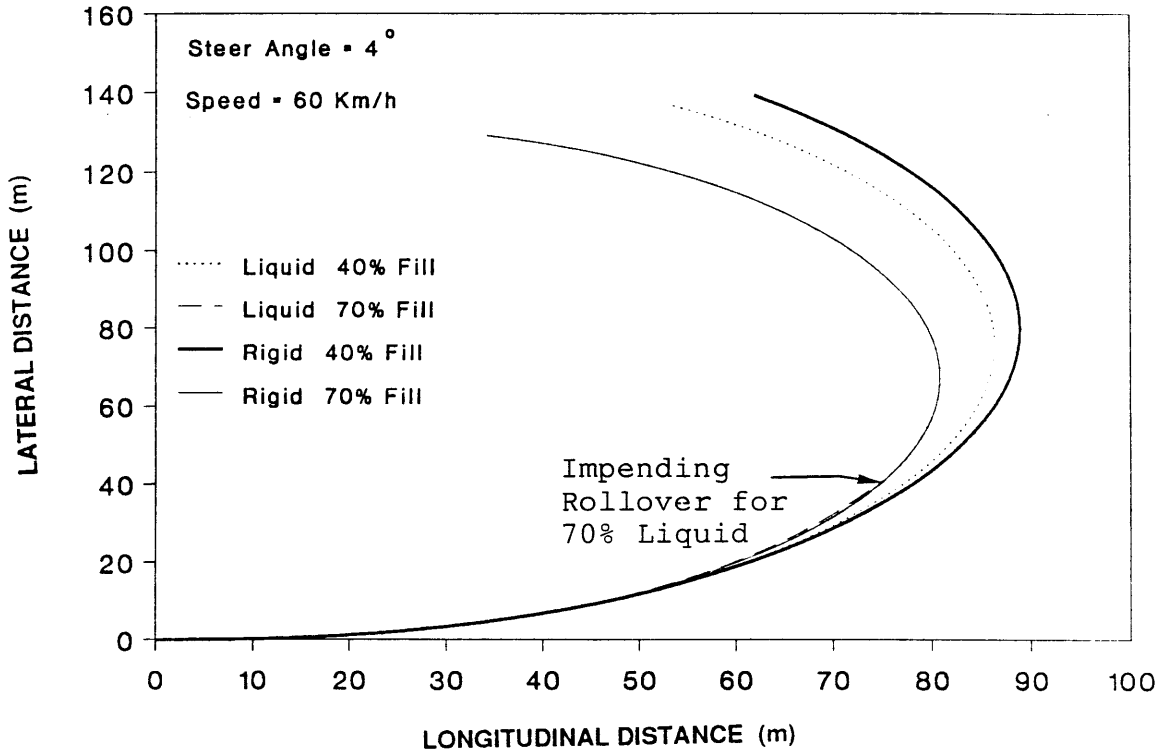


Fig. 8 Comparison of path followed by a partially filled tank vehicle and the equivalent rigid cargo vehicle subjected to steady steer input.

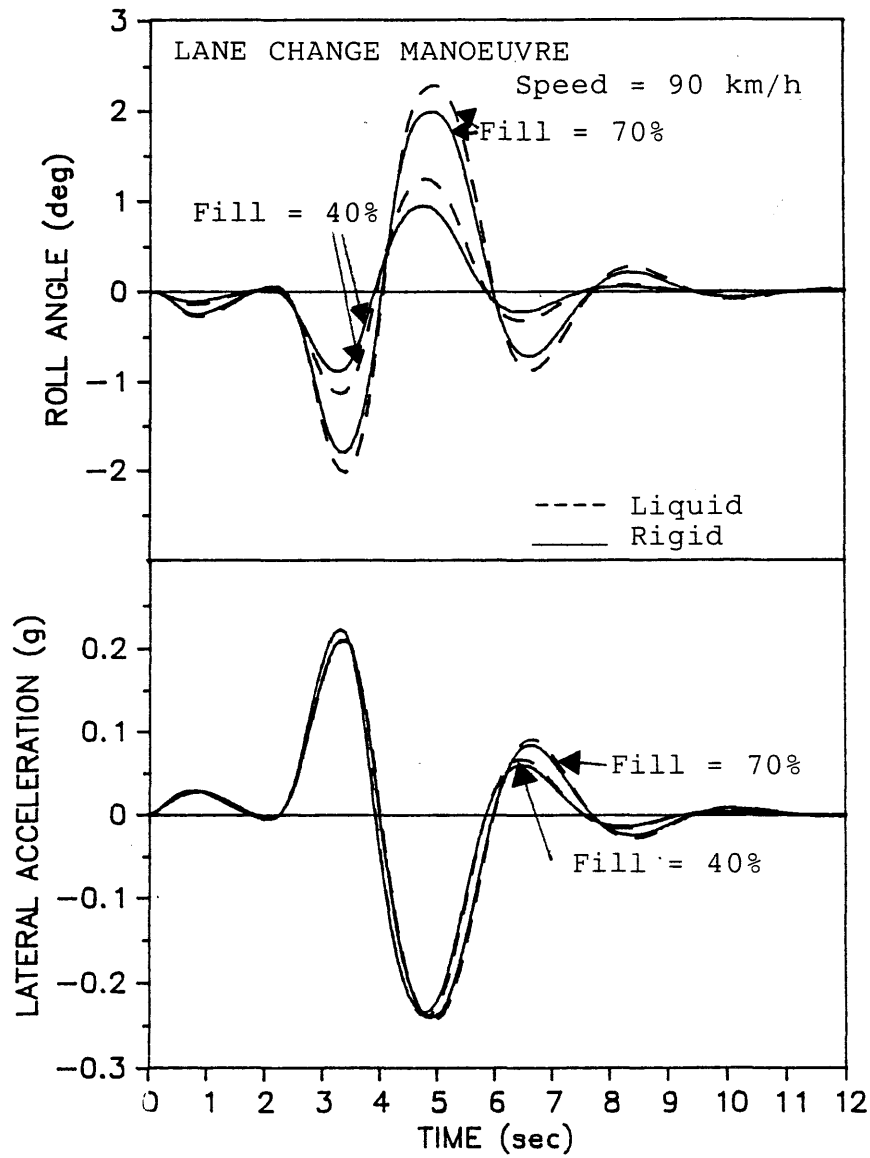


Fig. 9 Comparison of Dynamic Response of Tank Vehicle and Equivalent Rigid Cargo vehicle during a Lane Change Manoeuvre.



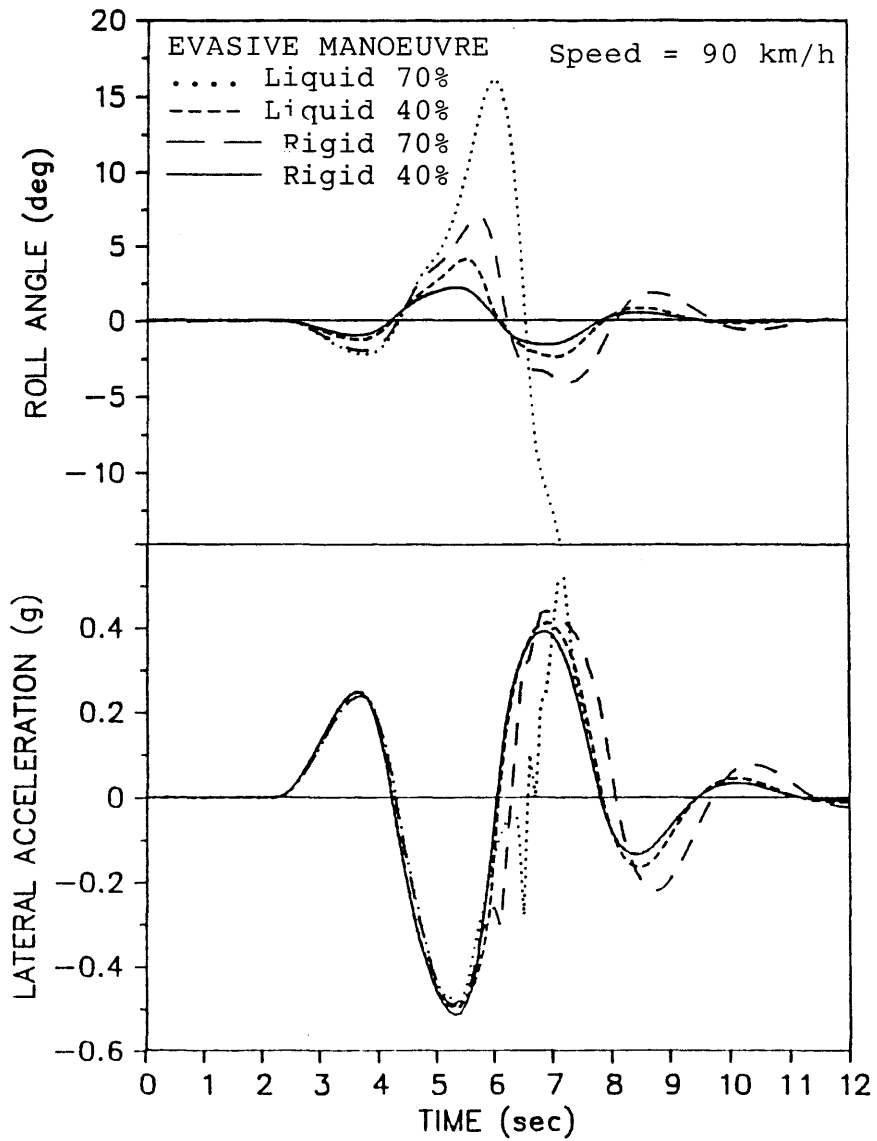


Fig. 10 Comparison of Dynamic Response of the Tank Vehicle and the Equivalent Rigid Cargo Vehicle during an Evasive Manoeuvre.

