# A perturbational testing method (PTM)

F. GYOKOS, Senior Staff Associate, Ministry of Labour, Hungary

Mechanical properties of tyres and decreasing smallest resonance frequency of compressed structural elements while their load increases call our attention to the importance of low frequency range within the vehicle/bridge interaction.
It is shown the common underlying principles of the lateral vibration and the lateral buckling in connection with a special beam-column.
The experimental results of application of PTM to this beam-column and tyres confirms the necessity of the dynamic approach of stability problems of structures working in the range of low frequencies.

## 1.Introduction

Considering the process of the vertical relative displacement of the bridge and the vehicle as an input mechanical influence on the tyre,we may look on the tyre forces as the output mechanical effect with a view to the tyre.
On the other hand the tyre forces are the input mechanical influence on both the bridge and the vehicle and causes different mechanical responses in their structure.
The tyre works as an intermediary element between the vehicle and the bridge.Their mutual effects are filtered because of mechanical transfer properties of the tyre.
Since the structure of the bridge,tyre and the vehicle transform the mechanical influences into some sort of output mechanical processes,it is possible to utilize the same theoretical considerations for chracterization of their mechanical condition.
In this approach it seems to be justified conception to characterize these "transformers" with an abstract function which expresses the relations between functions expressing the input and output time processes.
It is most convenient to determine the Fourier transform of the unit impulse response function [H(f)] of the tested phisical sytem experimentally.
PTM uses a small-scale random perturbation of the equilibrium of the tested structure.
PTM seems to be a suitable, sensitive method to show mechanical transfer properties of structures or structural elements and to show changes of transfer properties due to an increase of the load or the process of any degradation in the structure.
PTM provides K(f) and $\phi$ (f) as the magnitude and the phase of H(f) directly by real-time,dual-channel signal processing of electronically processable signals which represents the input and the output mechanical processes during the mentioned perturbation.
Technical facilities of up-to-date full-scale mechanical testing laboratories generally are suitable to execute the small-scale random perturbation of the equilibrium of the tested structure in a limited frequency range from zero Hz and to perform the necessary signal processing and Fourier transformations to obtain acceptable estimates of K(f) and $\phi$ (f).
If the tested system is linear then the H(f) characterises the system perfectly.
If the tested system is not linear then the H(f) characterises the system only close to that equilibrium state which was perturbed by the random,small value excitation.
At the end of eighties PTM was initiated in the full-scale Structural Laboratory of the Hungarian Institute for Building Science in Szentendre.
Amsler and MTS type, electronically controlled servo-hidraulic loading equipment driven by a random noise generator and a Bruel & Kjaer type dual-channel signal analiser was applied as the technical devices for PTM.
PTM proved to be a very sensitive and economical method in many cases of testing various loadbearing structures.
The results of testing for lateral buckling of beam-columns and testing agricultural spraying machines provided lessons which are worth considering for the study of vehicle bridge interaction.

## 2.Similarity between the elastic stability and free vibration of axially compressed bars

The similarity of elastic stability and free vibration of axially compressed bars manifests itself in the well known mathematical equations and formulas.
This similarity exists for plates ,too.
The paper of Harold Lurie (ref.1) summarizes this question.
His sequence of ideas is stimulating for further utilizing this similarity.
Striking analogies are shown when certain modifications are introduced.



*Heavy vehicles and roads: technology, safety and policy.* Thomas Telford, London, 1992.

The similar resonance and magnification factors, as

$$1/[1-(\frac{\omega}{\omega_0})^2] \text{ and } 1/[1-\frac{F}{F_0}]$$

refer to some connection between $F_0$ and $\omega_0$.
Here the the resonance factor is the multiplying factor for the harmonic shape of the deflection curve for a forced vibration with frequency $\omega$ related to $\omega_0$.
The magnification factor is the multiplying factor for the harmonic initial deflection shape of the compressed bar under F related to the shape of unloaded bar.
The $\omega_0$ is the frequency of the free vibration of the unloaded bar and $F_0$ is the static critical load of the bar.
A general equation is shown for bars, beams and plates in the following form :

$$\left(\frac{\omega}{\omega_n}\right)^2 = \frac{w L^*(u)}{u L(w)}$$

where L and $L^*$ are differential operators corresponding to beams or plates.
For example in the case of prismatic beams:

$$L(w)=\frac{\partial^4 w}{\partial z^4},\ L^*(u)=\frac{d^4 u}{dz^4}+F\frac{1}{K_y}\cdot\frac{d^2 u}{dz^2}$$

| | |
|---|---|
| w=w(z): | the mode of free vibration |
| u=u(z): | the mode of vibration while the beam is under axial compression(F) |
| $\omega_n$ | frequency of free vibration corresponding to the "n"th mode |
| $\omega$ | frequency of lateral vibration while the beam is under axial compression |

Supposed that $w=u$, in the case of a straight bar, the (1) general equation lead to the following linear expression:

$$\left(\frac{\omega}{\omega_n}\right)^2 = 1 - \frac{F}{F_n} \qquad (1)$$

It refers to the "n"th mode of vibration and buckling of the beam.
$F_n$ is the "n"th critical load, $\omega_n$ is the "n"th resonance frequency of the unloaded beam, F is thel compression load and the $\omega$ is the resonance frequency of lateral vibration of the axially compressed beam.
Fig.1 illustrates that the square of the frequency varies linearly with the axial force.
If the F tends to zero the resonance frequency tends to the natural frequency of free vibration.
At zero frequency F becomes the buckling load.
The $\omega_0$ and $F_0$ are the smallest resonance frequency of free vibration of the unloaded beam and the smallest statical critical load, otherwise the Euler's load.
The linearity of $\omega^2(F)$ function has importance for nondestructive testing structures for

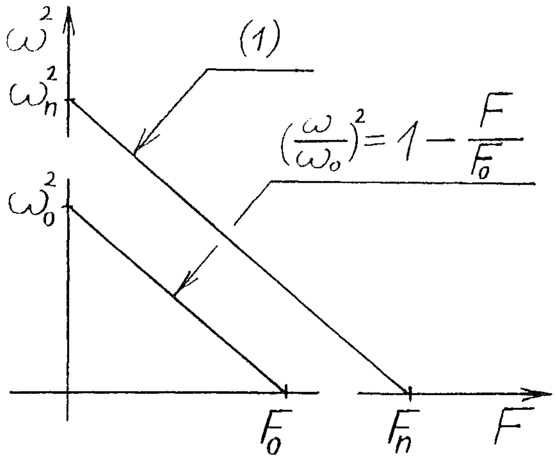


Fig.1

indirect experimental determination of the critical load.
If $w(z) \neq u(z)$ then the linearity of $\omega^2(F)$ function will no longer be valid.
The question arises as to the magnitude of the deviation from linearity.
This question arises for the beam-column, too, where the compression is not axial.

## 3. Dynamic concept of stability of beam-columns

Assuming a beam-column loaded at it ends in its simmetry plane, the basic equations of the equilibrium are the followings:

$$K_\omega \varphi^{IV} - K_c \varphi'' = m - K_p m'' \qquad (2)$$

$$K_y u^{IV} = f_x \qquad K_y = EJ_y \qquad (3)$$

where $K_\omega, K_c, K_p, K_y$ are stiffness quantities, $\varphi$ and $u$ are the rotation and lateral displacement of the beam-column, $\varphi^{IV}$, $\varphi''$, and $u^{IV}$ are their fourth and second derivatives.
The $m$ and $f_x$ are the distributed torsional moment and distributed lateral load on the beam-column.
Considering inertial forces in addition to effects due to eccentric force F

$$m = F d u'' - F\frac{J_x+J_y}{A}\varphi'' - \vartheta\ddot{\varphi}, \qquad (4)$$

$$f_x = -F u'' - F d\varphi'' - \rho\ddot{u} \qquad (5)$$

where "d" is the eccentricity of the force F.
$J_x, J_y, A$ are cross-sectional quantities,
$\vartheta$ and $\rho$ are the moment of inertia and mass per unit length of the beam-column, $\ddot{\varphi}$ and $\ddot{u}$ are second derivatives as the accelerations of $\varphi$ (the rotation) and $u$ (lateral displacement of the axis of the beam-column). The $m$ and $f_x$ are intensities of the distributed torsional moment and the lateral load on the beam-column.
Introducing relations (4) and (5) into equations (2) and (3), the basic system of equilibrium equations are the followings:

$$\varphi^{IV}\left(K_\omega - FK_p\frac{J_x+J_y}{A}\right) + \varphi''\left(F\frac{J_x+J_y}{A} - K_c\right) +$$

$$+u^{IV}K_p d\cdot F - u'' dF -$$

$$-K_p \nu \ddot{\varphi}'' + \nu \ddot{\varphi} = 0 \qquad (6)$$

$$K_y u^{IV} + Fu'' - Fd\varphi'' + \rho\ddot{u} = 0 \qquad (7)$$

$$K_\omega = \frac{EJ_\omega}{K}, \quad K_c = GJ_c$$

$$K_p = \frac{K_\omega}{GJ_p} \qquad K = \frac{J_p - J_c}{J_p}$$

These equations are valid in the case of neglecting the damping.

### 3.1 The static case

The (6) and(7) equations become simpler in the static case as

$$\varphi^{IV}\left(K_\omega - FK_p\frac{J_x+J_y}{A}\right) + \varphi''\left(F\frac{J_x+J_y}{A} - K_c\right) +$$

$$+u^{IV}K_p dF - u'' dF = 0 \qquad (8)$$

$$K_y u^{IV} + Fu'' - \varphi'' dF = 0 \qquad (9)$$

If the eccentricity d=0 then (10) and (11) basic equations issue from (8) and (9).

$$\varphi^{IV}\left(K_\omega - FK_p\frac{J_x+J_y}{A}\right) + \varphi''\left(F\frac{J_x+J_y}{A} - K_c\right) \qquad (10)$$

$$K_y u^{IV} + Fu'' = 0 \qquad (11)$$

If $\varphi(0)=0$, $\varphi(\ell)=0$, $\varphi''(0)=0$ and $\varphi''(\ell)=0$ are the boundary conditions for straight prismatic bar the smallest axial torsional critical load F results from the (10)equation.

$$F_\varphi = \frac{K_c + K_\omega\frac{\pi^2}{\ell^2}}{\left(1 + K_p\frac{\pi^2}{\ell^2}\right)\frac{J_x+J_y}{A}} \qquad (12)$$

The "$\ell$" is the length of the compressed beam.the special stiffness feature of tested beam-columns are expressed by this particular form of $F_\varphi$ These special features follows from nature of the special shape of cross-section of the thin-walled I-beam with hollow flanges and unstiffened web.

The classical Euler formula (13) results from the (11) equation in the case of a simple supported axially compressed straight bar.

$$F_y = K_y\frac{\pi^2}{\ell^2} \qquad (13)$$

The loss of stability of equilibrium of an axially compressed beam proceeds at $F_y$ and at $F_\varphi$ independently,even at $F_x$ ,too,if the x and y axes are the so-colled main axes of the cross-section.

$$F_x = K_x\frac{\pi^2}{\ell^2} \qquad (14)$$

If $K_x > K_y$ and d=0 then the critical load( $F_c$ ),at which the eccentrically compressed beam-column loses the stability of its equilibrium,results from the(8) and (9) equations taking into account the actual boundary conditions.

If the boundary conditions are

$$u(0)=0, \; u''(0)=0, \; \varphi(0)=0, \; \varphi''(0)=0$$

Then the "critical load" is the following:

$$F_c = \frac{F_y + F_\varphi - \sqrt{(F_y+F_\varphi)^2 - 4F_yF_\varphi\left(1 - \frac{A}{J_x+J_y}d^2\right)}}{2\left(1 - \frac{A}{J_x+J_y}d^2\right)} \qquad (15)$$

### 3.2 Natural vibration of unloaded beams

Substituting F=0 into (6) and (7) results the following equations which are the basic equations of determination of natural frequencies,otherwise resonance frequencies of the unloaded beam.

$$K_\omega\varphi^{IV} - K_c\varphi'' = -\nu\ddot{\varphi} + K_p\nu\ddot{\varphi}'' \qquad (16)$$

$$K_y u^{IV} = -\rho\ddot{u} \qquad (17)$$

Taking into account the boundary conditions of the simple supported beam,the square of the smallest torsional resonance frequency $\alpha^2$ and the square of the smallest resonance frequency for lateral vibration $\beta^2$ yield in the following form:

$$\alpha^2 = \frac{\left(K_c + K_\omega\frac{\pi^2}{\ell^2}\right)\frac{\pi^2}{\ell^2}}{\left(1 + K_p\frac{\pi^2}{\ell^2}\right)\nu} \qquad (18)$$

$$\beta^2 = \frac{1}{\rho}K_y\frac{\pi^4}{\ell^4} \qquad (19)$$

Compared (18)and (19) with (12) and(13) appear the connections between resonance frequencies of the unloaded beam and the static critical loads of the axially compressed beam.

$$\alpha^2 = \frac{1}{\varrho}\,\frac{\pi^2}{\ell^2}F_\varphi \qquad (20)$$

$$\beta^2 = \frac{1}{\varrho}\,\frac{\pi^2}{\ell^2}F_y \qquad (21)$$

3.3 The general case of the eccentrically compressed beam- column

On the basis of general basic equations (6) and (7) follows the general relation between the square of the smallest resonance frequency ($\omega^2$) of free lateral vibration of eccentrically compressed beam-column and the eccentric load (F)

$$F_yF_\varphi\frac{\pi^4}{\ell^4} - \omega^2\varrho\left(F_y+F_\varphi\right)\frac{\pi^2}{\ell^2} +$$

$$(\omega^2\varrho)^2 - F\left(F_y+F_\varphi\right)\frac{\pi^4}{\ell^4} +$$

$$+F^2\left(1-\frac{A}{J_x+J_y}d^2\right)\frac{\pi^4}{\ell^4} = 0 \qquad (22)$$

Using results of 3.1 and 3.2 this relation can be written in an equivalent form to it as follows:

$$(\varrho\beta^2)(\varrho\alpha^2) - \omega^2\varrho\nu\left(\beta^2+\alpha^2\right) +$$

$$+\omega^4\varrho\nu - F\nu\left(\beta^2+\alpha^2\right)\frac{\pi^2}{\ell^2} +$$

$$+F^2\left(\frac{J_x+J_y}{A}-d^2\right)\frac{\pi^4}{\ell^4} = 0 \qquad (23)$$

Similarly to the (22) and (23) equivalent expressions can be given for the critical load ($F_c$) and for the resonance frequency ($\omega_0$) of the unloaded beam-column:

$$\frac{\pi^2}{\varrho\ell^2}F_c = \frac{\beta^2+\alpha^2-\sqrt{\left(\beta^2+\alpha^2\right)^2-4\beta^2\alpha^2\left(1-\frac{A}{J_x+J_y}d^2\right)}}{2\left(1-\frac{A}{J_x+J_y}d^2\right)} \qquad (24)$$

$$F_c = \frac{F_y+F_\varphi-\sqrt{\left(F_y+F_\varphi\right)^2-4F_yF_\varphi\left(1-\frac{A}{J_x+J_y}d^2\right)}}{2\left(1-\frac{A}{J_x+J_y}d^2\right)} \qquad (25)$$

$$\frac{\varrho\ell^2}{\pi^2}\omega_0^2 = \frac{F_y+F_\varphi\pm\sqrt{\left(F_y+F_\varphi\right)^2-4F_yF_\varphi}}{2} \qquad (26)$$

$$\omega_0^2 = \frac{\beta^2+\alpha^2\pm\sqrt{\left(\beta^2+\alpha^2\right)^2-4\beta^2\alpha^2}}{2} \qquad (27)$$

The latest two expressions lead to

$$\left(\omega_0^2\right)_1 = F_\varphi\frac{\pi^2}{\varrho\ell^2} = \alpha^2 \qquad (28)$$

$$\left(\omega_0\right)_2^2 = F_y\frac{\pi^2}{\varrho\ell^2} = \beta^2 \qquad (29)$$

expressions.
These mathematical relations and formulas express the inherency of phenomenas of the buckling of statically,eccentrically compressed beam-column and its lateral vibration.

3.4 Results of computations

Thin-walled beam-columns was tested in the Structural Laboratory of the Hungarian Institute for Building Science in Szentendre.
Fig.2 showes the sketch of the specimen and its loading and supporting arrangement.

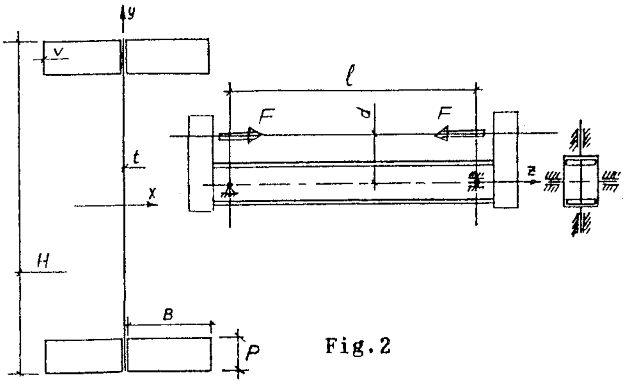


Fig.2

LOAD-FREQUENCY~2 DIAGRAM 01-02

01:open flanges 02:hollow flanges

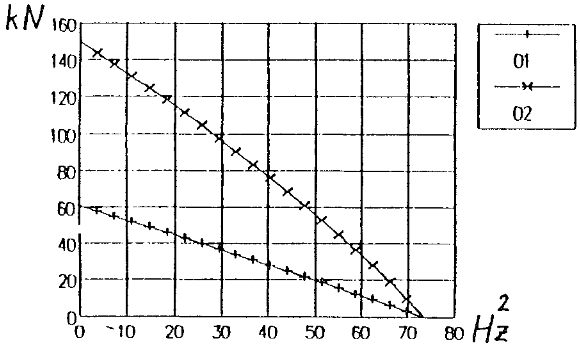


Fig.3

LOAD-FREQUENCY~2 DIAGRAM 3- 4

3: open flanges 4: hollow flanges

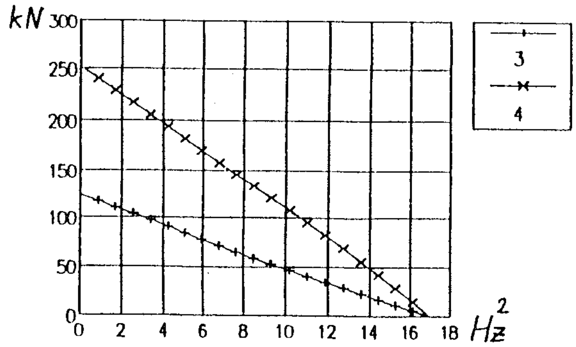


Fig.4

Fig.3 and Fig.4 represent results of computations on the basis of (22) and(23) equations.

Beam-columns with open flanges and hollow flanges was compared by their $\omega^2(F)$ functions.The deviation from the linearity of the $\omega^2(F)$ functions in spite of the extreme eccentricity (d) proved to be small.

Data of beam-columns marked as 01-02:
l=5600mm,d=700mm,H=560mm,B=60mm, P=40mm, v=3mm, t=2mm

Data of beam-columns marked as 3-4:
l=9600mm,d=600mm,H=600mm,B=100mm P=50mm v=3mm, t=5mm

The big difference between results of beam-columns with open and hollow flanges comes about big differences between their torsional stiffness.If the four-sided rectangular thin-walled tubes are opened,practically they are not welded lengthways in the middle of their inner side of the flange (see Fig.2),the upper and bottom flanges lose their relativly big torsional stiffness.

## 4.PTM for testing mechanical conditions of beam-columns under different eccentric compression

An MTS -type electronically controlled servohidraulic actuator (250 kN,hub:160mm) provided F which was increased step by step.

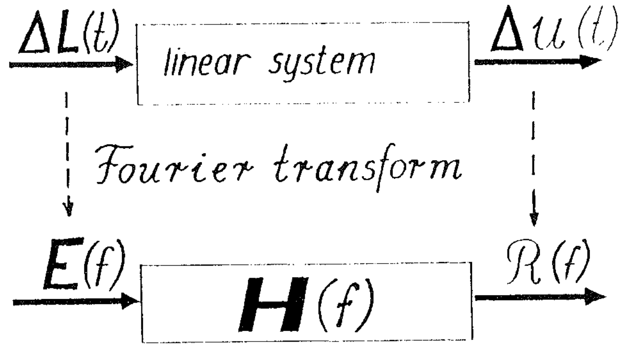


Fig.5

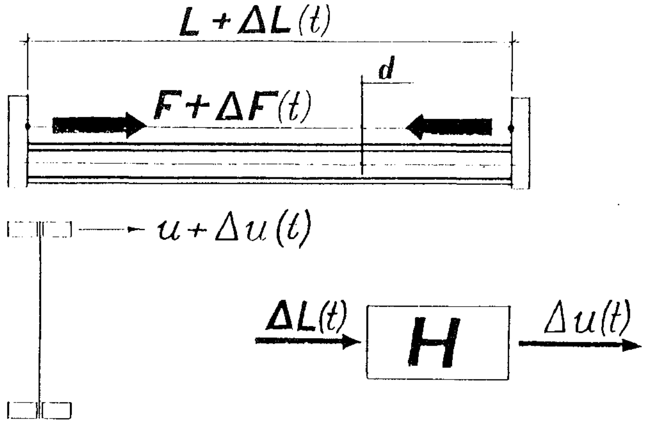


Fig.6

At all particular static F the equilibrium of the beam-column was perturbed with a small random moving of the cylinder of the same actuator.

The small-scale random moving of the cylinder was controlled by signals of a noise generator. The perturbating displacement of the actuator was considered as an excitation.The lateral displacement of the upper flange of the beam at the cross- section in the middle of the beam was considered as the response of the excited specimen.

A real time, Bruel and Kjaer-type two-chanel signal analiser performed the correlation analizis of the input (excitation) and the output (response) processes.

Results of the real-time signal- processing was utilized to determine resonance frequencies for the experimental $\omega^2(F)$ function.

Fig.5 and Fig.6 are the schemes of the performed test utilizing PTM.

Since the perturbation was a small-scale one and in such a way the beam-column remained close to its equilibrium state,it is justified to suppose practically that the beam-column behaves as a linear system during the perturbation of its particular equilibrium state.

H(f) is the complex "frequency response function"

The change of value of $K(\nu)$ seems to be asuitable quantity to characterize the change of the stability of the tested equilibrium of the tested steel beam-columns.

$K(\nu)$ is a particular value of the K(f) function which is one of direct results of the utilized PTM as the magnitude of H(f).

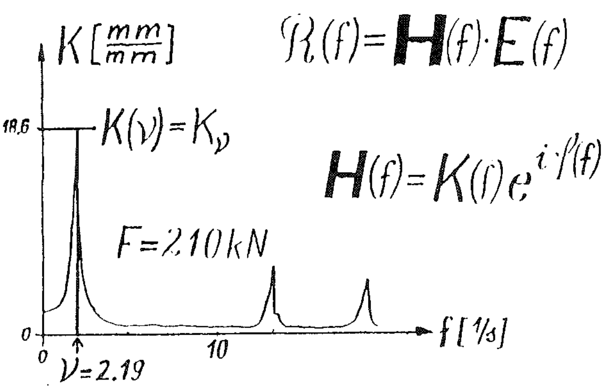


Fig.7

As it is shown in the Fig.7 the    is the smallest experimentally determined resonance frequency at a particular level of the static load(F).
Fig.8 and Fig.9 shows the experimental K(f) fuctions at F=160kN and F=210kN for beam-columns marked as"3-4"(see above).
Theoretical and experimental results are compared in the Fig.10 and the different values of $K(\nu)=K_\nu$ are shown at different levels of the eccentric compression of the tested beam-column.
The acceleration of the change of the $K(\nu)$ close to the critical load is particularly remarcable.

## 5. Lessons

The case of the studied beam-columns is a very symple example compared to structures of bridges and the vehicles.
After all the sketched sequences of ideas are valid for structures of bridges and vehicles and the PTM is a suitable method to utilize results of the theoretical dynamics and the up-to-date technical facilities as the product of high-technology.
Figures from Fig.11 to Fig.16 confirm the necessity of the dynamic approach of stability problems of structures,particularly the bridge and vehicle structures.
Fig.11-Fig.13 showw K(f) of a tyre at three levels of its load.
This tyre is a part of an agricultural spraying machine.
Fig.14 showes K(f) of a tyre of a lorry.
Fig.15 gives K(f) functions of a model of a prestressed concrete highway bridge.
Values of the smallest resonance frequencies can be seen.
Data of Fig.11-Fig.16 show,that undesirable coincidence may happen.

The fig.10 has an important lesson.The static and the additional random load work in the symmetry plane of the beam-column.They do not give reasons for lateral vibration of the beam in principle.But the load probably does not work exactly in the symmetry plane and the beam probably has initial geometric imperfections.These are reasons of the undesirable lateral vibration.The Fig.10 shows well the accelerated increasing of the undesirable effect of the unavoidable intial imperfections close to the loss of stability.

On the other hand these phenomena are very suitable to utilize the PTM for characterization of the actual mechanical condition of the tested structure.

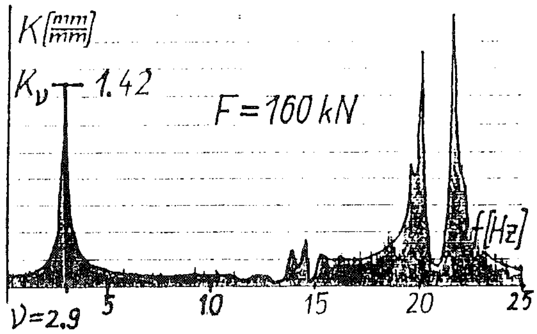


Fig.8

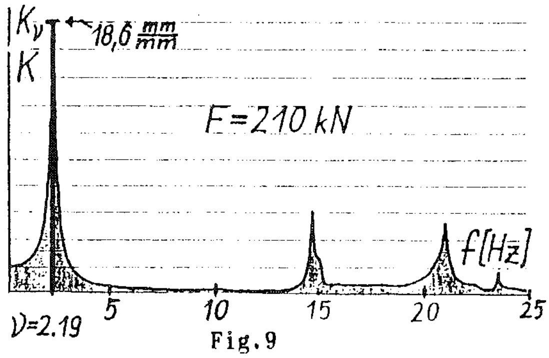


Fig.9

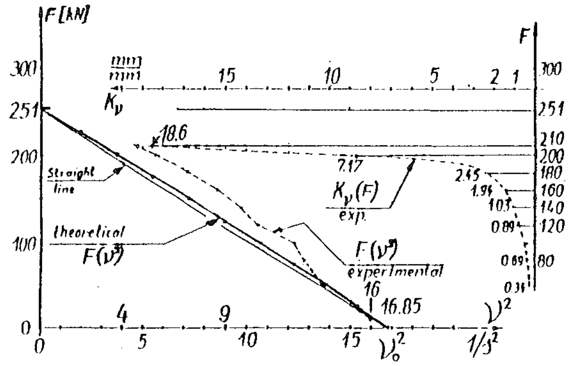


Fig.10

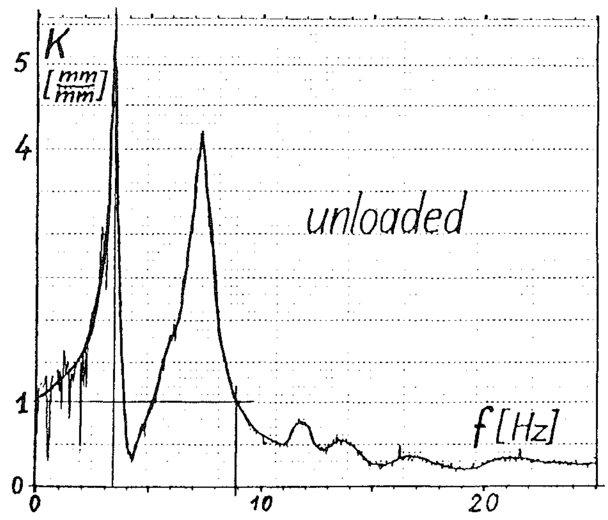


Fig.11

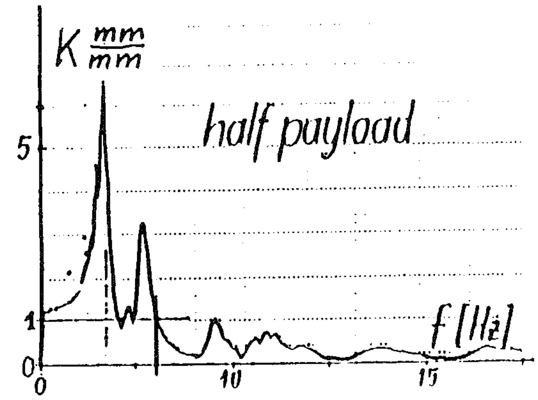


Fig.12

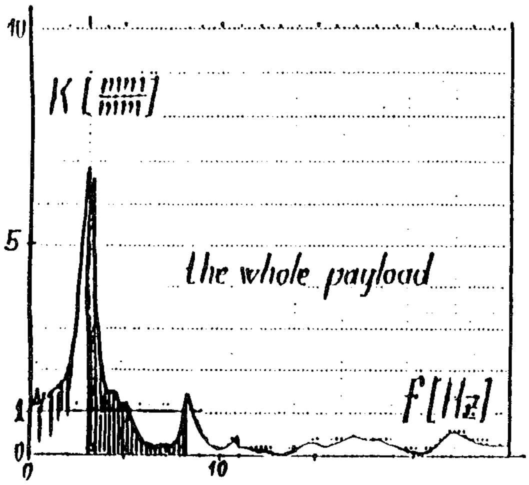


Fig.13

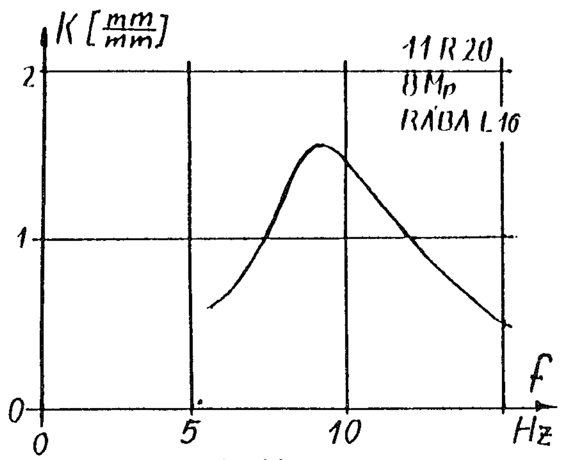


Fig.14

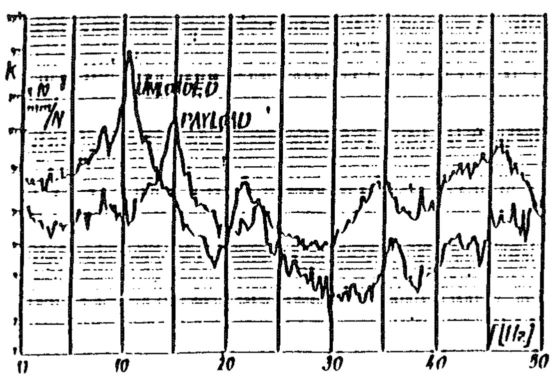


Fig.15

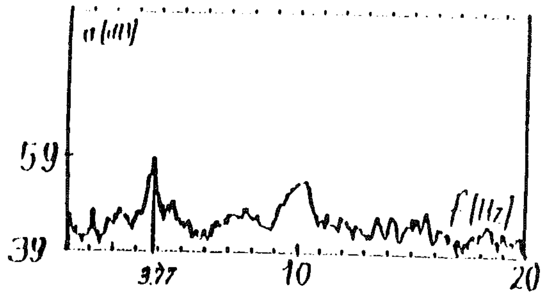


Fig.16


REFERENCES

1.LURIE H. Journal of Applied Mechanics June 1952,195-204 Lateral Vibrations as Related to Structural Stability

2.SOUZA M.A.Eng. Struct.,1987,Vol.9,April Post-buckling vibration characteristic of structural elements

3.TIMOSHENKO S.,YOUNG D.H.,WEAWER W.JR.: Vibration Problems in Engineering p.521.John Wiley and Sons,4th ed.

4.ILLESSY J.Testing Possibilities of Degradation of Bridges (Hungarian) 21.552/80,ETI (Hungarian Institute for Building Science)

5.MIHALFFY P.Dr. Hungarian Research and Development Company for the automotive Industry dissertation and Testing document (KO2U-17/80)



Acknowledgement

I owe Dr.Mihalffy and eng.Illessy thanks for their permissions for presentation of their figures in this paper as Fig.14 and Fig.16 .