

Quadratic evaluation of the stability domain and experimental validation of the behaviour of an articulated vehicle

M. KHAN, Research Assistant, Department of Mechanical Engineering, P. BOURASSA, Professor, Department of Mechanical Engineering, B. MARCOS, Professor Department of Chemical Engineering, G. PAYRE, Professor, Department of Mechanical Engineering, and S. RATTE, Undergraduate student, Department of Mechanical Engineering, Sherbrooke University, Quebec, Canada

For linear systems, simple and efficient methods of analysis are readily available as well as analytical solutions and the local behaviour of these systems may be extended to the global system behaviour. On the other hand, the local behaviour of non linear systems may never be generalized to the global state space and the analytical solution of the equations of motion are rarely available. Also, there exist no universal principle under which non linear systems may be treated and which may claim a successful analysis of any of these systems.

In this work, the dynamic stability of non linear system representing tractor semitrailer vehicles is studied on the base of a qualitative problem approach. Starting with a small restricted stability domain obtained by using the Hamiltonian of the system as an initial Lyapunov function, and following a direct search procedure with the unconstrained minimization technique (S.U.M.T.), the stability domain of this vehicle system was successfully maximized for quadratic type Lyapunov functions and for the proposed search method.

1. INTRODUCTION:

Tractor semitrailer have caused a large number of accidents and often with many resulting death and serious injuries. Consequently, applied research has been addressed to this sector of activities and in particular to the area of simulation and dynamic stability. Over the last decades, many studies were subsidized for the purpose of identifying the phenomenon of dynamic instability and the system parameter that leads to such instability, such as jackknifing and snaking.

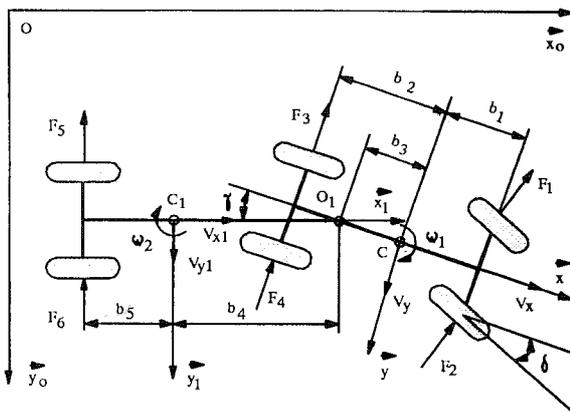
Results of studies implying linear models (Jindra [1963a, 1963b], Hales [1963], Ellis [1966] and schmid [1967]) have been confirmed experimentaly results so long as perturbations which occurred around the stable equilibrium point were of small amplitudes. Meanwhile these linear models were found totally deficient with respect to non oscillatory divergent type of motion, jackknifing, and also for of the divergent oscillatory motion known as snaking.

Mikulcik [1968, 1971] was one of the first investigators to adequately model the nonlinear system of truck semitrailer and account for jackknife following step steer or braking manoeuvres. Among other important contributions, are Ellis [1963], Susmihl [1972] for identification and indirect sensing of tractor jackknifing, Ervin and Yoram [1986] for the influence of the dimensions and weights of these vehicles on their dynamical performance, and for their sensitivity to such variations. Eshleman and al. [1973] have presented one the most elaborate study in this field. In their investigation, the dynamic behaviour of these articulated vehicles was predicted through the use of numerical integration and the Lyapunov method of the first kind. They favourably

compared their finding with experimental measurements. They also compared various methods for generating Lyapunov functions.

A third group of studies deals with the entire domain of stability of these vehicles. Olusola and Linkins [1975], used Lyapunov's second method to study the stability of a vehicle moving on a straight road. Later Sachs and Chou [1976], employed the same method to establish the stability domain of an automobile. Singh [1978, 1980] presented one of the most interesting study based on the second method of Lyapunov. He used the Hamiltonian of the system as a quadratic Lyapunov function and obtained a domain of stability, which turn out meanwhile to be smaller than the domain established through numerous and lengthy numerical integration of the motion equations within the whole state space following the try and error strategy.

In this particular study, a search strategy is devised through which a particular quadratic Lyapunov function is determined that will maximize the size of the stability hypervolume without having to perform the very time costly procedure of integrating the motion for the numerous cases of parameter sets. The actual adopted strategy and the type of Lyapunov function were suggested through the results obtained by Eshleman and al [1973]. In their study, they claim that second Lyapunov method represents the best available technique for studying the road vehicles stability by a non linear model. Among the different schemes for generating the Lyapunov functions, they consider that the quadratic estimate and the Zubov integration as best fitted for a numerical optimization of the domain of attraction. On the other hand, they state that the quadratic estimate leads to better result than Zubov's method beeing more precise and also taking less time to converge.



$$\begin{aligned}
 M_T &= 6803 \text{ kg.} & M_S &= 29483 \text{ kg.} & I_T &= 19697 \text{ kg.m}^2 \\
 I_S &= 422235 \text{ kg.m}^2 & b_1 &= 1.56 \text{ m} & b_2 &= 1.91 \text{ m} \\
 b_3 &= 1.55 \text{ m} & b_4 &= 6.27 \text{ m} & b_5 &= 3.66 \text{ m.}
 \end{aligned}$$

Figure 1: Articulated vehicle model.

2. MODELING:

In the present model, (figure 1), the tractor and semitrailer units are articulated at the fifth wheel through a vertical axis. The tractor is assumed to travel at a constant forward velocity on a plane uniform horizontal road with a constant radius of curvature. Generalized coordinates involved are the tractor forward motion, its lateral displacement and the yaw angle of both units. Pitch and roll angles are neglected. Alignment torques and forces resulting from multiple wheels pairs and multiple axle bogies are not considered in the present analysis.

3. EQUATIONS OF MOTION:

To facilitate the writing of the motion equations, two cartesian coordinate systems, $(C, \vec{x}, \vec{y}, \vec{z})$ and $(C_1, \vec{x}_1, \vec{y}_1, \vec{z}_1)$, are attached to the tractor and the semitrailer respectively.

Applying Newton's second law and the theorem of the angular momentum to each unit of figure 1, and accounting for the 5th wheel constraints, the following differential equations of motion can be easily derived:

$$\begin{aligned}
 (M_T + M_S)\dot{V}_Y - M_S b_3 \dot{\omega}_1 - M_S b_4 \dot{\omega}_2 \cos \gamma = \\
 -(M_T + M_S)V_X \omega_1 + M_S b_4 \omega_2^2 \sin \gamma + (F_1 + F_2) \cos \delta \\
 + (F_3 + F_4) + (F_5 + F_6) \cos \gamma
 \end{aligned} \quad (1)$$

$$\begin{aligned}
 -M_S b_3 \dot{V}_Y + (I_T + M_S b_3^2) \dot{\omega}_1 + M_S b_3 b_4 \dot{\omega}_2 \cos \gamma = \\
 M_S b_3 V_X \omega_1 - M_S b_3 b_4 \omega_2 \sin \gamma + b_1 (F_1 + F_2) \cos \delta \\
 - b_2 (F_3 + F_4) - b_3 (F_5 + F_6) \cos \gamma + C_f
 \end{aligned} \quad (2)$$

$$\begin{aligned}
 -M_S b_4 \dot{V}_Y \cos \gamma + M_S b_3 b_4 \dot{\omega}_1 \cos \gamma + (I_S + M_S b_4^2) \dot{\omega}_2 \\
 = M_S b_4 V_X \omega_1 \cos \gamma - M_S b_4 V_Y \omega_1 \sin \gamma + M_S b_3 b_4 \omega_1 \sin \gamma \\
 - (b_4 + b_5)(F_5 + F_6) - C_f
 \end{aligned} \quad (3)$$

Where the articulation angle between the longitudinal axis of both units may be obtained through the integration of:

$$\frac{d\gamma}{dt} = \dot{\gamma} = \omega_1 - \omega_2 \quad (4)$$

Forces F_i , called the side slip forces, are build up at the interface between the road and the tyre. They either balance external forces applied to the axles or else the inertia forces encountered during the lateral motion of the vehicle. These forces arises while a certain contact area will develop between the tyre and the road and a slip angle will be encountered when they are applied. The relationship between the lateral forces F_i , the vertical load W_i and the slip angle α_i for any tyre will depends on the tyre elastic structure, the shape of the contact zone, the state of the road wear, the coefficient of friction and many other factors. Many empirical formulas have been proposed to compute these forces. A very good formula approximating the real tyre behaviour has been proposed by Ellis [1969], expressing the lateral forces as a non linear cubic type polynomial in term of the side slip angle as follows:

$$F_i = C_{i1} \alpha_i + C_{i3} \alpha_i^3 \quad (5)$$

where the coefficients C_{i1} and C_{i3} are taken as constant.

In this actual study, a modified formula proposed by Eshleman & al. [1973], was preferred which accounted for measurements obtained with Hi-Miller 10.00-20 tyres. This formula reads as follows:

$$F_i = \frac{-1.5 \mu_i Z_i}{\alpha_{mi}} \left(\alpha_i - \frac{\alpha_i^3}{\alpha_{mi}^2} \right) \quad (6)$$

where

$$\mu_i = 1. - \frac{0.35 Z_i}{30258}. \quad (7)$$

and

$$\alpha_{mi} = 0.1557079 \left(1. + \frac{Z_i}{30258} \right) \quad (8)$$

where, for each tyre i , Z_i represent the quasi-static vertical loading and α_i , the slip angle expressed as follows:

$$\alpha_1 = \alpha_2 = \frac{V_Y + b_1 \omega_1}{V_X} - \delta \quad (\text{front tractor wheels}) \quad (9)$$

$$\alpha_3 = \alpha_4 = \frac{V_Y - b_2 \omega_1}{V_X} \quad (\text{rear tractor wheels}) \quad (10)$$

and for the semitrailer wheels:

$$\alpha_5 = \alpha_6 = \frac{V_X \sin \gamma + (V_Y - b_3 \omega_1) \cos \gamma - (b_4 + b_5) \omega_2}{V_X \cos \gamma - (V_Y - b_3 \omega_1) \sin \gamma} \quad (11)$$

Assuming there is no anti-jackknife device at the fifth wheel, ($C_f = 0.$), that is, there is no motion restraint around the axis of articulation between the two units, then the general motion equations (1)-(4), with the equations can be written as follows:

$$[A]\{\dot{X}\} = \{F(X)\} \tag{12}$$

where

$$[A] = \begin{bmatrix} (M_T + M_S) & -M_S b_3 & -M_S b_4 \cos \gamma & 0 \\ -M_S b_3 & (I_T + M_S b_3^2) & M_S b_3 b_4 \cos \gamma & 0 \\ -M_S b_4 \cos \gamma & M_S b_3 b_4 \cos \gamma & (I_S + M_S b_4^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\{X\} = (V_Y, \omega_1, \omega_2, \gamma)^T$$

and where

$\{F(X)\} = (F_1(X), F_2(X), F_3(X), F_4(X))^T$ are the right hand members of equations (1) to (4) in this order. Since the determinant of the matrix $[A]$ is always positive, $[A]^{-1}$ exist and (12) is transformed into:

$$\{\dot{X}\} = [A]^{-1}\{F(X)\} \tag{13}$$

4. STABILITY OF THE VEHICLE SYSTEM:

In this section, two aspects of the articulated vehicles will be treated (a) the stability in a close neighbourhood of the equilibrium points and (b) the determination of the boundary of the domain of attraction around a stable equilibrium point.

The solution of the equation $\{F(X)\} = \{0\}$ for $\{X\}$ in R^4 yields an infinity of solutions that is an infinity of dynamic equilibrium points commonly called "singular points" for the system described by equation (13).

In order to determine the vehicle state in the neighbourhood of one of the equilibrium points $\{X_e\}$, a Taylor expansion of the equation set (13) is performed in the following form:

$$\{\dot{Y}\} = [B]\{Y\} + \{H(Y)\} \tag{14}$$

where

$$\{Y\} = \{X\} - \{X_e\}$$

and $[B]$ is the jacobian matrix of the system (13) evaluated at the considered equilibrium point $\{X_e\}$. According to the first Lyapunov method, the system would be stable in the neighbourhood of $\{X_e\}$ if and only if $[B]$ possesses eigenvalues with negative real parts. A list of some equilibrium point of the system (13) appear in table 1. They have been computed for a longitudinal velocity of 75 km/h, a steering angle of the front wheels of 3° and for the state vector $\{X\}^T$ in the range $\{-10., -10., -10., -\pi\}^T$ and $\{10., 10., 10., \pi\}^T$. The numerical integration of the equations (13) in the neighbourhood of the points II and III of table 1 has made it possible to represent the behaviour of the system in the state space of the vehicle with respect to small perturbation around a stable and unstable point of equilibrium. The results appear in figure 2 and 3.

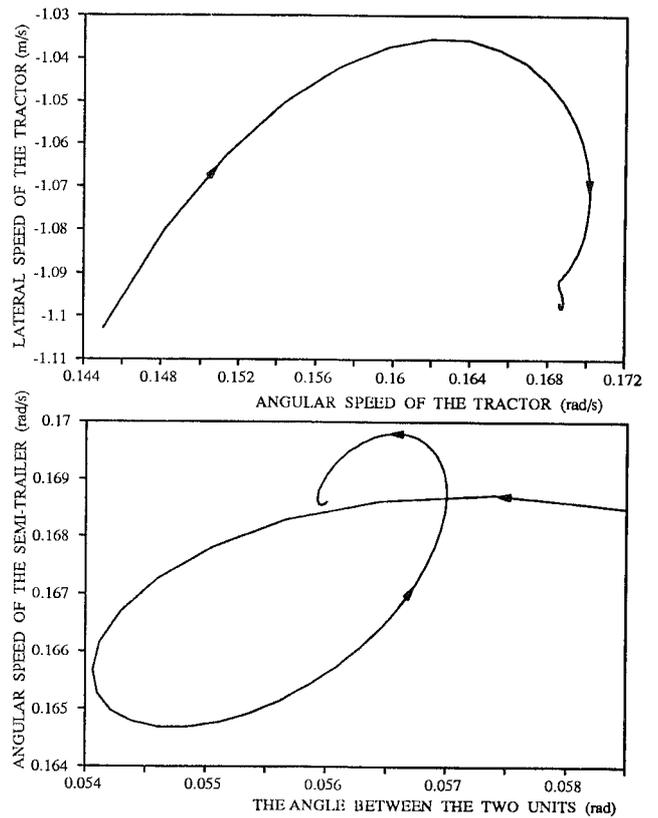


Figure 2: Trajectories of motion projected on (V_y, ω_1) and (ω_2, γ) in the neighbourhood of a stable point.

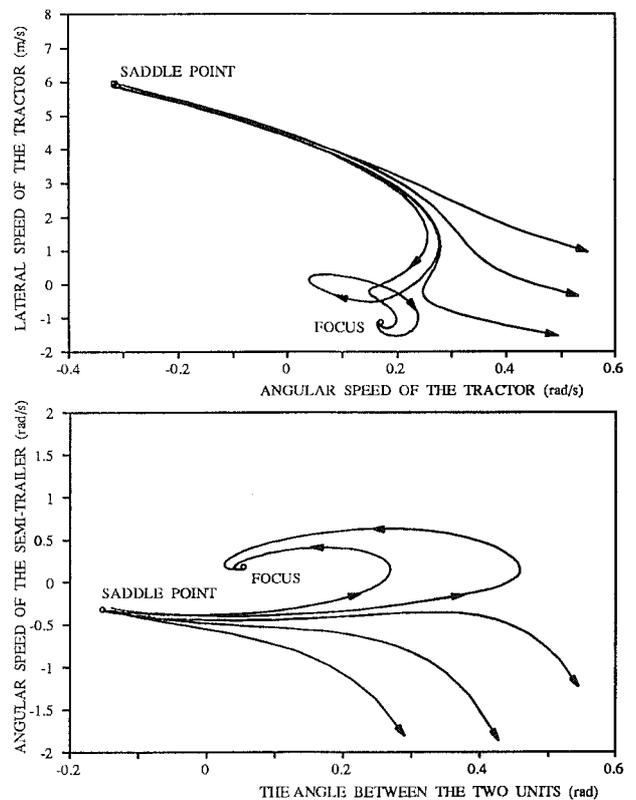


Figure 3: Trajectories of motion projected on (V_y, ω_1) and (ω_2, γ) in the neighbourhood of an unstable point.

4.1 **DOMAIN OF STABILITY:**

The ensemble of perturbation $\{X\}$ for which the system trajectories return to a close neighbourhood of the equilibrium state represent the domain of asymptotic stability. Apart from the numerical integration process which turns out to be a trial and error strategy process, a very time consuming one indeed, as soon as the number of number of independent coordinates exceeds two, there exist no other systematic method that will determine the boundary of the attraction domain. Accordingly, many of investigators have used the second method of Lyapunov to determine the extent of the stability domain. This method consist in the search of a scalar positive definite function $V(Y)$, called Lyapunov function, with continuous partial derivatives inside a domain Ω which contains the equilibrium point $\{Y_e\} = \{0\}$ and such that:

$$V(Y) > 0 \quad \forall \{Y\} \neq \{0\} \in \Omega \quad (15)$$

$$V(0) = 0 \quad (16)$$

$$\dot{V}(Y) < 0 \quad \forall \{Y\} \neq \{0\} \in \Omega \quad (17)$$

$$\dot{V}(0) = 0 \quad (18)$$

For the purpose of this investigation, a quadratic function of the following type was selected:

$$V(Y) = \{Y\}^T [P] \{Y\} \quad (\text{with } [P] \text{ symmetric}) \quad (19)$$

where the corresponding domain of vehicle stability is given according to the above theorem by:

$$\{Y\}^T [P] \{Y\} < V_{\min} \quad ; \quad V_{\min} > 0 \quad (20)$$

THEOREM: (Shields [1973])

Let E_v be the set

$$E_v = \{ \{Y\} / \dot{V}(Y) = 0. ; \{Y\} \neq \{0\} \}$$

then the region of asymptotic stability indicated by the Lyapunov function $V(Y)$ of (19) is given by D where:

$$D = \{ \{Y\} / V(Y) < V_{\min} \}$$

and where:

$$V_{\min} = \min(V(Y)) \quad ; \quad \{Y\} \in E_v$$

The derivative of $V(Y)$ is given by:

$$\dot{V}(Y) = \{\dot{Y}\}^T [P] \{Y\} + \{Y\}^T [P] \{\dot{Y}\} \quad (21)$$

Replacing (14) into (21) and taking into account that $[P]$ is symmetric, it follows:

$$\dot{V}(Y) = -\{Y\}^T [Q] \{Y\} + 2\{Y\}^T [P] \{H(Y)\} \quad (22)$$

with:

$$[Q] = -([B]^T [P] + [P] [B]) \quad (23)$$

From the preceding section, it is known that the system (14) is stable in the neighbourhood of point III of table 1. It may be therefore be concluded that the time derivative of the Lyapunov function, $\dot{V}(Y)$ is negative definite in at least a small confined region around the equilibrium point $\{Y_e\} = \{0\}$. This result may be confirmed by taking $[Q]$ positive definite and $\{Y\}$ small enough to bring the quadratic term in (22) predominant. Therefore the conditions (16) to (18) are verified. The condition (15) is ipso facto satisfied since the selection of $[Q]$ as a positive definite matrix implies the positive definiteness of $[P]$ (The inverse proposition does not hold)(see Lasalle & Lefschitz [1961]).

Therefore, for every symmetric positive definite matrix $[Q]$, the problem consist in solving (23) for $[P]$ and subsequently minimizing the following function:

$$\left. \begin{array}{l} \min(V(Y)) \\ \text{subject to} \\ \dot{V}(Y) \leq 0 \end{array} \right\} \quad (24)$$

The solution of this constrained problem can be obtained with the help of the sequential unconstrained minimization technique with which an estimate can be achieved for the domain of stability corresponding to $[Q]$ and whose volume is proportional to that of the hypervolume $\{Y\}^T [P] \{Y\} = V_{\min}$ which is given by:

$$Vol = \pi \left(\frac{V_{\min}^4}{\text{Det}(P)} \right)^{\frac{1}{2}} \quad (25)$$

Then, in order to maximize the stability domain, it is sufficient to generate a set of positive definite matrix $[Q]$ and to select the particular one which yield the largest volume (25) while satisfying the conditions (24).

4.1.1 **Algorithm for generating the s.d.p. matrices $[Q]$:**

A simple way would consist in randomly generating matrices and selecting among them those which pass the test of positive definiteness. This procedure is very time costly and yields no useful information for optimizing the search and is therefore simply not efficient. Then the following procedure has been taken in this investigation. The matrix $[Q]$ is written as a matrix product as follows:

$$[Q] = [L][L]^T \quad (26)$$

where $[L]$ is a lower triangular matrix. From this expression, the volume (25) may be maximized over the field of the $[L]$ matrices rather than the field of $[Q]$

Table 1: List of some singular points.

Sing. point	Coord. of the sing. point	Eigenvalues of matrix $[B]$	System state
I	-9.128, -3.854 -3.854, -1.344	2.593±i(11.388) 11.506, 2.525	unstable
II	5.884, -0.319 -0.319, -0.149	5.849, -4.721 -0.708, 1.769	unstable
III	-1.103, 0.168 0.168, 0.056	-2.560±i(3.209) -1.195±i(1.471)	Stable
IV	-1.457, 0.187 0.187, -0.230	-2.633±i(2.466) 4.519, -1.146	unstable

matrices. Indeed the positive definiteness of $[Q]$ matrices is guaranteed for all non singular matrix $[L]$. Starting with the matrix:

$$[L] = \begin{bmatrix} L_1 & 0 & 0 & 0 \\ L_2 & L_3 & 0 & 0 \\ L_4 & L_5 & L_6 & 0 \\ L_7 & L_8 & L_9 & L_{10} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (27)$$

the vector $\vec{Q} = (L_1, L_2, \dots, L_{10})^T$ and the incrementation bases $B(\vec{e}_1, \vec{e}_2, \dots, \vec{e}_{10})$ and $B_1(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_{10})$ are formed and an increment step P_0 is selected. The components e_{ij} and u_{ij} of the respective vectors \vec{e}_i and \vec{u}_i are given by:

$$e_{ij} = 1 \quad (\text{for } j \leq i) \quad \text{and} \quad e_{ij} = 0 \quad (\text{for } j > i)$$

$$u_{ij} = 1 \quad (\text{for } j = i) \quad \text{and} \quad e_{ij} = 0 \quad (\text{for } j \neq i)$$

Then the procedure continues as follows:

1°) For $i=1$, the volume (25) is evaluated for $(\vec{Q}-P_0\vec{e}_1), \vec{Q}$ and $(\vec{Q}+P_0\vec{e}_1)$ in order to determine the direction in which the incrementation of \vec{Q} will increase the volume estimate of stability domain. Subsequently \vec{Q} is incremented in this direction with the step of $P_0\vec{e}_i$ until the best value \vec{Q}_1 is reached, that is, when the incrementation process can no longer improve the value of the volume (25).

2°) This procedure is iterated, taking $\vec{Q} = \vec{Q}_{i-1}$ ($[L]=[L_{i-1}]$), letting i vary from 2 to 10 in order to obtain the vector \vec{Q}_0 (the matrix $[L_0]$) which maximize the volume (25) with the step P_0 .

3°) Halving P_0 , and taking $\vec{Q} = \vec{Q}_0$ as a starting vector, steps 1 and 2 are repeated, yielding a new vector \vec{Q}_0 corresponding to a maximum volume of (25) for P_0 and $P_0/2$.

4°) The procedure is brought to a stop when the incrementation step falls below a minimum preestablished value P_{min} .

5°) Repeating steps 1 to 4 with the incrementation base vectors \vec{u}_i of B_1 , \vec{Q}_{max} ($[L_{max}]$) was finally reached which maximized the volume (25) for the selected search method.

5. RESULTS:

The domain of stability obtained with this procedure is shown in figure 4 in the space (V_y, ω_1, γ) . The multi-dimension volume is now represented by plane intersections. In figure 4, are shown intersections of this volume with plane $\gamma = \text{constant}$ that is, for articulation angle γ .

On figure 5, the stability domains for $\gamma = 3.23^\circ$ are shown as obtained with three different approaches i) with numerical integration of motion equations ii) the initial domain as obtained with the Lyapunov method for $[Q] = [I]$ and iii) The final volume as obtained with the above maximization procedure.

It is worth insisting here, that to each of the constant angle γ of domain on figure 4, the yaw velocity of the semitrailer and of the tractor have been set equal.

6. EXPERIMENTAL VALIDATION:

The theoretical study is currently validated with small scales articulated physical models such as shown in figure 6. Small scale experimentation definitely brings a lot of very interesting advantages; first, these physical models are fairly inexpensive. Second, there is little concern for safety. Jackknifing may be produced repeatedly with no harm to the model or even the instrumentation. Third, a large variety of tyres are available for standard type models and the tyre properties can be measured quite accurately on small inexpensive dynamometer benches. Also, these tyre measurements are carried fairly rapidly covering linear and non linear range of the slip angle up to the maximum friction ellipse, and for a range of normal loading. For the lateral dynamic validation, data such as inertia, dimensions and masses are obtained readily. In full scale experimentation, one has to rely on scarce published tyre and vehicle data.

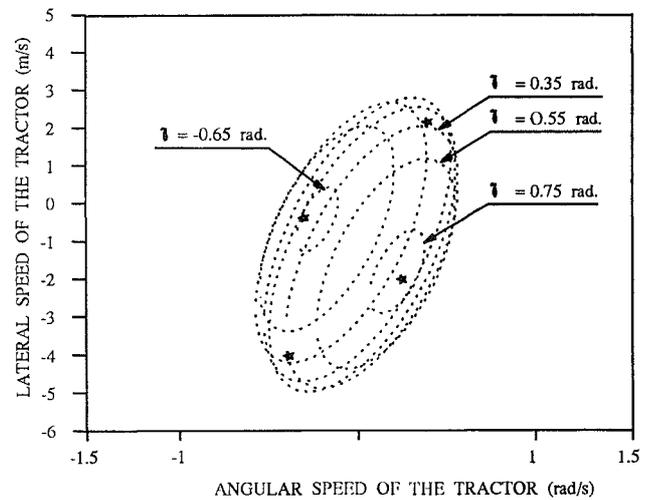


Figure 4: Domain of stability in (V_y, ω_1, γ) space.

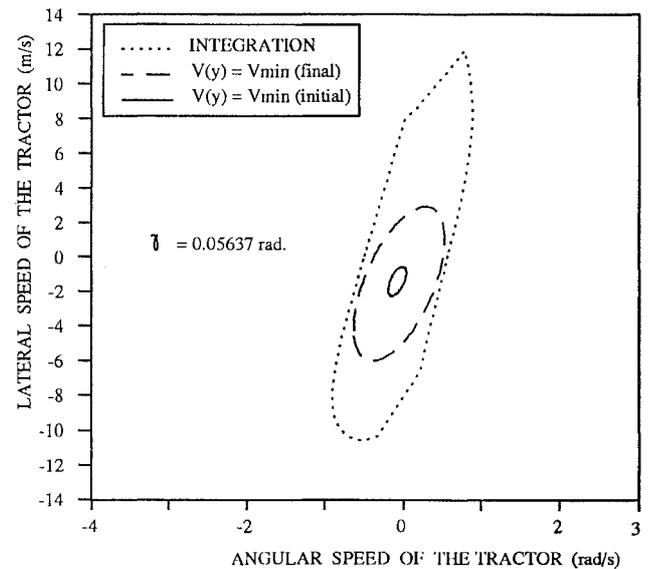


Figure 5: Stability domain section ($\gamma = 3.23^\circ$).

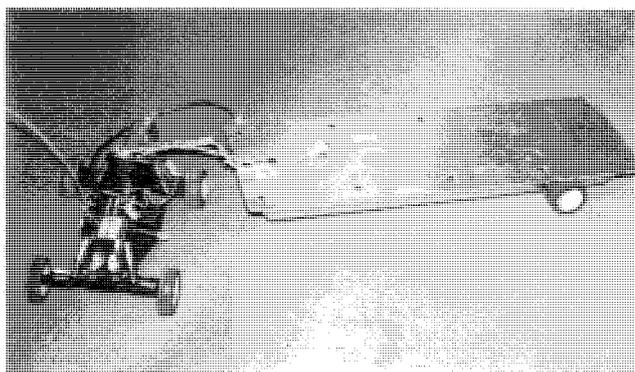


Figure 6: Small scale experimentation vehicle.

ACKNOWLEDGMENTS:

This investigation has been made possible through grants from Transport Canada and by a grant of the Quebec Research Foundation F.C.A.R. 91 TM-1032.

BIBLIOGRAPHY:

- ELLIS, J., R. [1963] The Dynamics of Vehicles During Braking. Proceeding of Symposium on Control of Vehicles during Braking and Cornering, London, England.
- ELLIS, J., R. [1966] The Ride and Handling of Semi-Trailer Articulated Vehicles. *Automobile Engineer*, pp. 523-529.
- ELLIS, J., R. [1969] *Vehicle Dynamics*. Business Books Limited, London UK.
- ESHLEMAN, R., L.; DESAI, S., D.; D'SOUZA, A., F. [1973] Stability and Handling of Articulated Vehicles, part 1. Technical Report IITRI, Chicago, Illinois.
- FIACCO, A., V.; McCORMICK, G., P. [1966] Extension of SUMT for Non-Linear Programming: Equality Constraints and Extrapolation. *Management Science*. Vol 12 # 11, pp. 816-828.
- HALES, F., D. [1963] The Lateral Stability of a Simplified Articulated Vehicle. Proceedings of Symposium on Control of Vehicles during Braking and Cornering, London, England.
- JINDRA, F. [1963 a] Tractor and Semi-Trailer Handling. *Automobile Engineer*, pp. 438-446.
- JINDRA, F. [1963 b] Lateral Stability of Three Unit Trailer Train. *Oester. Ing. Arch.* Vol 17, pp. 209-226.
- LASALLE, J.; LEFSCHITZ, S. [1961] *Stability by Lyapunov's Direct Method with Applications*. Academic Press, N.Y., London.
- MIKULCIK, E., C. [1968] The Dynamics of Tractor-Semitrailer Vehicles: The Jackknifing Problem. Ph.D Thesis, Cornell University.
- MIKULCIK, E., C. [1971] The Dynamics of Tractor-Semitrailer Vehicles: The Jackknifing Problem. SAE Paper # 710045.
- MOORING, B., W.; GENIN, J. [1982] A Kinematic Constraint Method for Stability Analysis of Articulated Vehicles. *International Journal Of Vehicles Design*. Printed in U.K. Vol 3 # 2.
- OLUSOLA, O., B.; LINKINS, P., W. [1975] Stability Analysis of Constant-Span-Transit Vehicles on Straight Horizontal Fixed Guideways. Proceedings of the Third International Conference on Vehicle System Dynamics. Swets and Zeitlinger, B. V., Amsterdam.
- SACHS, H., K.; CHOU, C., C. [1976] On the Stability in the Sense of Lyapunov of Rubber Tyre Vehicle. *Transactions of the ASME, Journal of Dynamic Systems, Measurement and Control*. Vol 98 pp. 180-185.
- SCHMID, I. [1967] Engineering Approach to Truck and Tractor Train Stability. SAE Paper # 670006
- SHIELDS, D., N., [1973] Ph.D Thesis, Department of Mathematics Loughborough University of Technology, U.K.
- SINGH, M. [1978] On the Theory of Dynamic Stability of Motor Vehicle Carriers. Ph.D Thesis, Wayne State University, Detroit, Michigan.
- SINGH, M. [1980] Stability Bounds of a Tractor-Semitrailer Vehicle System Dynamics. Vol 9, pp. 69-86.