

Super-singles: implications for design

A. M. IOANNIDES, PhD, Assistant Professor of Civil Engineering, University of Illinois, Urbana, USA, R. A. SALSILLI, PhD, Assistant Professor, Universidad de Chile, Instituto de Investigaciones y Ensayes de Materiales (IDIEM), Santiago, Chile, I. VINDING, MS, Head, Concrete Road Activities, Aalborg Portland, Copenhagen, Denmark, and R. G. PACKARD, PhD, Manager, Paving and Transportation Department, Portland Cement Association, Illinois, USA

An algorithm is proposed for the edge bending stress in a slab-on-grade concrete pavement under complex loading and boundary conditions using a personal computer spreadsheet. The method relies on dimensional analysis for data interpretation and overcomes the limitations imposed by several of Westergaard's assumptions. The procedure is used to reproduce the "Equivalent Stresses" in the *1984 PCA Guide*, and subsequently to investigate the effects of replacing conventional truck dual tires by a "super-single" tire. The significant detrimental effect of "super-singles" is clearly observed. A sensitivity analysis of the effect of a number of other variables is also presented.

1 INTRODUCTION

In the development of mechanistic pavement design procedures, it is highly desirable to establish algorithms that describe the structural response of pavements under a number of different loading and support conditions. Analytical solutions, such as those developed by Westergaard (ref. 1), have been routinely used for this purpose, but their applicability has often been limited by the restrictive assumptions made in their derivation (ref. 2). More realistic solutions may be obtained using a number of numerical techniques, such as the finite element method. Powerful though they may be, such computerized schemes cannot be incorporated easily in a design guide, primarily in view of the numerous executions that would be required for any given design objective. The ideal design algorithm would be one that could be expressed in a simple chart or nomogram, table or equation. It is also considered desirable that design algorithms be suited for application on a personal computer, such as might be encountered in a typical pavement design office.

This Paper presents an illustration of how dimensional analysis may be combined with statistics to establish a mechanistic design algorithm. The formulae obtained have been implemented in a personal computer spreadsheet, and may be used to determine quickly and reliably the edge bending stress occurring in a concrete pavement slab-on-grade, subjected to multiple-wheel edge loading. The proposed algorithm is verified by reproducing the "Equivalent Stresses" that form the basis for the U.S. Portland Cement Association (*PCA*) design procedure against fatigue (ref. 3). It is subsequently applied in investigating the effects of replacing conventional truck dual tires by a "super-single" tire, a practice that has become increasingly popular in recent years both in Europe and in North America (ref. 4). The significant detrimental effect of "super-singles" (ref. 5) is clearly demonstrated. A parametric study into the effect of several other variables is also presented.

2 DIMENSIONAL ANALYSIS AND STATISTICS IN PAVEMENT DATA INTERPRETATION

The development of robust design algorithms typically involves two important steps:

- (i) The compilation of a reliable database consisting of calculated or measured responses from numerous individual pavement systems; and
- (ii) The interpretation of such data, leading to the establishment of broad and meaningful conclusions, applicable to a wide spectrum of pavement systems, including many not considered in step (i).

Pavement engineers have used a number of different approaches in realizing these two goals. Thus, databases compiled have consisted of observations made on full scale, in-service pavements, or of results of laboratory tests, or even of outputs from repeated executions of selected computer codes. At the data interpretation stage, statistical analysis techniques have been employed, often with little recognition of the engineering interactions occurring between a host of individual input parameters. The limitations of an exclusively statistical data interpretation approach have been discussed in earlier publications (refs 6, 7). In the last five years, investigations at the University of Illinois and elsewhere have resulted in the formulation of a distinct data interpretation approach grounded upon the principles of dimensional analysis (refs 8, 9). This idea has received considerable attention among pavement engineers (refs 10-13), and the progress achieved through its application in many areas has been very encouraging.

The main contribution of dimensional analysis is that it helps identify the governing independent variables driving system response. These are usually expressed in the form of dimensionless ratios or products, sometimes also referred to as 'clusters' (refs 10, 12), and reflect engineering interactions among several input parameters. Dimensionless variables may be employed in subsequent statistical analyses, leading to the development of more fundamental and general

descriptions of the phenomena observed. Thus, dimensional analysis and statistics may be complementary in their application to the solution of pavement problems. The desirability of a synergistic approach to pavement data interpretation has recently received considerable support from prominent statisticians, as well. In the words of Hunter and Pendelton (ref. 14), for example: "The world of transportation science often employs mathematical models that go far beyond simple statistical constructions: models that entrain the laws of engineering and physics, non-linear in their parameters and often dynamic. Successful inference requires a blending of engineering knowledge and statistics. Enlightened empiricism accompanies good science."

3 DEVELOPMENT OF PROPOSED EDGE STRESS PREDICTION ALGORITHM

The concept of employing dimensional analysis in extending the applicability of available closed-form solutions (ref. 2) is hardly novel. Two classical examples with which most pavement engineers are familiar are the extension of Boussinesq's solution to a two-layered system by Burmister (ref. 15), and the extension of Burmister's solution itself to a multi-layered system by Odemark (ref. 16).

Burmister's extension was derived on the basis of a rigorous mathematical derivation and is, therefore, an "exact" general solution in the theory of elasticity. On the basis of elegant mathematical manipulations, Burmister showed that the behavior of a two-layered system was uniquely defined by two dimensionless ratios, namely (h/a) and (E_1/E_2) , where h and E_1 are the thickness and elastic modulus of the upper layer, respectively, E_2 is the elastic modulus of the supporting half-space, and a is the radius of the applied load. Burmister was, therefore, able to express the maximum deflection, Δ , in such a system as the product of the available Boussinesq solution for the homogeneous half-space multiplied by a "correction factor," F_w :

$$\Delta = 1.5 \frac{pa}{E_2} F_w \left[\frac{h}{a}, \frac{E_1}{E_2} \right] \quad (1)$$

Burmister presented a chart for the determination of F_w as a function of the two governing dimensionless ratios. Note that Eq. (1) may be recast as:

$$\frac{\Delta}{1.5pa} = F_w \left[\frac{h}{a}, \frac{E_1}{E_2} \right] \quad (2)$$

which is in the form of:

$$R^* = f[\pi_i, \pi_j] \quad (3)$$

where R^* : dimensionless response; f : function sought; and π_n : governing dimensionless products or ratios.

It should be recalled that the celebrated Westergaard equations may also be expressed in a dimensionless form (ref. 6):

$$R^* = f[\pi_k] \quad (4)$$

The simplification afforded by plate theory in the analysis of concrete pavements becomes immediately

apparent since Eq. (4) involves only one dimensionless ratio ($\pi_k = a/l$), compared to two in Eq. (3). Here, l denotes the radius of relative stiffness of the slab-subgrade pavement system.

Odemark's approach is outwardly different from Burmister's method, but accomplishes the same results: extension of the applicability of an available theoretical solution to a more complex system. Rather than continuing Burmister's mathematical derivations, which become extremely complex when more than two layers are considered, Odemark introduced his own ingenious, if simplified and approximate, "Method of Equivalent Thicknesses" (MET). Furthermore, in deriving a solution for a three-layer system, Odemark could have introduced, after Burmister, additional correction factors. He opted, however, to derive an "equivalent" modulus, E_m , which characterizes an imaginary material replacing the last constructed layer and the natural subgrade. Repeated application of E_m reduces any multi-layered system to Burmister's two-layer idealization, or even to Boussinesq's half-space. For the case of a three-layer concrete pavement system, Odemark showed that an adequate approximation for E_m is given by:

$$E_m = E_3 \left[1 + 1.08 \left[\frac{h_2}{h_1} \right]^3 \sqrt{\left[\frac{E_2}{E_1} \right]} \sqrt{\left[\frac{E_m}{E_3} \right]} \right] \quad (5)$$

which is of a form very similar to that of Eq. (3):

$$\left[\frac{E_m}{E_3} \right] = F_7 \left[\frac{E_1}{E_2}, \frac{h_2}{h_1} \right] \quad (6)$$

In these expressions, subscripts 1 and 2 denote the upper and lower constructed layers, while subscript 3 refers to the natural subgrade. Odemark also presented a chart for function F_7 in terms of the two pertinent dimensionless ratios, thus permitting the continued use of Eq. (1), with E_2 set equal to E_m .

The lessons learnt from such a reexamination of these two classical works can be very useful to pavement engineers involved in data interpretation activities. In fact, it can be argued that the questions confronting the profession today in accomplishing step (ii), above, are often more difficult than those pertaining to step (i), i.e., that "the problem is no longer one of data availability, but one of data interpretation" (ref. 6). The approaches followed by Burmister and Odemark suggest that if we were able to identify the controlling dimensionless ratios, π_n , we could use our databases to define either additional "correction factors" or mechanistically formulated "equivalent" quantities. Engineering mechanics and dimensional analysis are invaluable in identifying π_n and formulating the required "equivalent" quantities. For its part, statistics can be employed in providing "best fit" descriptions to the function f sought in each case.

Previous investigations (refs 2, 8) have led to significant breakthroughs in the identification of governing dimensionless parameters for a variety of complex pavement problems of practical interest. In a project conducted at the University of Illinois, Salsilli

(ref. 17) employed stepwise regression analysis techniques using the SAS software package (ref. 18) to establish relations between dimensionless parameters identified in earlier studies and the edge bending stress, σ , occurring in a slab-on-grade subjected to multiple-wheel loading along one of its edges (see Table 1). Expressed as a dimensionless response in accordance to Eq. (4), this stress can be written as $(\sigma h^2/P)$, where h is the concrete slab thickness and P is the total applied load (refs 19, 6). The database used by Salsilli consisted primarily of a small number of *ILLI-SLAB* finite element results, illustrating the significant savings in the quantity of data required to reach broad and meaningful conclusions. This efficiency is achieved when statistics is not relied on exclusively in data interpretation, but is used merely as a curve-fitting tool, after engineering mechanics and dimensional analysis concepts have been exploited to the maximum extent practical. Most of the formulae in Table 1 can be considered as multiplicative "correction factors" which are applied sequentially to Westergaard's prediction, σ_{West} (refs 1, 20). The purpose of each factor is to eliminate one of Westergaard's restrictive assumptions. Thus, the procedure adopted for the development of these factors is akin to the methodology followed by Burmister, except for the fact that numerical rather than analytical results were used in this investigation.

The application of the concept of the "Equivalent Single Area" (*ESAR*) for the accommodation of multiple-wheel loads in Table 1 deserves a further explanation. The basic idea is already evident in a paper by Bradbury (ref. 21), and was suggested recently (ref. 6) as a means of addressing the limitations of two concepts that have been used extensively in pavement design. These are the statistical/empirical Equivalent Single Axle Load (*ESAL*) (ref. 22), and the more mechanistic Equivalent Single Wheel Load (*ESWL*) (ref. 23). The *ESAR* concept imposes no *a priori* assumptions as to the total applied load, contact pressure or size of tire print, and leads to a reasonably precise estimation of the maximum stress under a multiple-wheel load. This is achieved through the use of Westergaard's equations for a single-wheel load, into which the equivalent radius, a_{eq} , of a multiple-wheel assembly is substituted. Thus, the application of the *ESAR* concept is akin to Odemark's MET.

4 EDGE STRESS PREDICTION IN THE PCA GUIDE

The proposed stress prediction algorithm consisting of the formulae in Table 1 may be verified by reproducing the "Equivalent Stresses", σ_{eq} , given in Tables 6a and 6b of the *1984 PCA Guide* (ref. 3). A brief description of the derivation of these stresses is presented first. The basic database used in the development of the "Equivalent Stress" Tables consisted of results obtained using the *J-SLAB* finite element computer program for the input parameters shown in Table 2a. On the basis of these results, a number of regression equations were developed, relating the edge moment, M_e , to the pavement radius of relative stiffness, l . These equations are shown in Table 2b, and include

Table 1. Proposed Formulae [After Salsilli(ref.17)]

Westergaard (1948) Edge Stress Equation:

$$\sigma_{West} = \frac{3(1+\mu)P}{\pi(3+\mu)h^2} \left[\ln \frac{Eh^3}{100ka^4} + 1.84 - \frac{4\mu}{3} + \frac{1-\mu}{2} + 1.18(1+2\mu)(a/l) \right]$$

Alternate Formula for $(a/l) > 0.5$:

$$\frac{\sigma_{corr}}{\sigma_{West}} = 1 - 0.0621(a/l)^2 + 0.131(a/l)^3$$

Equivalent Radius for Duals, Spacing S (Perpendicular to Edge):

$N=20$; $R^2=1$; $COV=1.2\%$. Limits: $0 < (S/a) \leq 20$; $0.05 \leq (a/l) \leq 0.5$

$$\frac{a_{eq}}{a} = 0.909 + 0.339485(S/a) + 0.103946(a/l) - 0.017881(S/a)^2 - 0.045229(S/a)^2(a/l) + 0.000436(S/a)^3 - 0.301805(S/a)(a/l)^3 + 0.034664(S/a)^2(a/l)^2 + 0.001(S/a)^3(a/l)$$

Equivalent Radius for Tandems, Spacing t (Parallel to Edge):

$N=16$; $R^2=0.997$; $COV=2.1\%$. Limits: $4 \leq (t/a) \leq 16$; $0.05 \leq (a/l) \leq 0.3$

$$\frac{a_{eq}}{a} = 2.199479 + 0.74761 \ln(t/a) \ln(a/l) + 0.548071 \ln^2(t/a) - 0.486597 \ln^2(t/a) \ln(a/l) - 0.29507 \ln^3(t/a) - 0.028116 \ln^3(a/l)$$

Effect of Axle Width D (Perpendicular to Edge):

$N=28$; $R^2=0.995$; $COV=6.9\%$. Limits: $0.13 \leq (D/l) \leq 3$; $0.05 \leq (a/l) \leq 0.3$

$$\frac{\sigma_D}{\sigma_{West}} = -0.15743211 + 0.26935303(a/l) + 0.357644(l/D) - 0.0589073(l/D)^2 + 0.003486(l/D)^3$$

Effect of Slab Size, Length L (Parallel to Edge):

$N=12$; $R^2=0.996$; $COV=0.29\%$. Limits: $3 \leq (L/l) \leq 5$; $0.05 \leq (a/l) \leq 0.3$

$$\frac{\sigma_L}{\sigma_{West}} = 0.582282 - 0.533078(a/l) + 0.181706(L/l) - 0.019824(L/l)^2 + 0.109051(a/l)(L/l)$$

Effect of Load Transfer Efficiency, Aggregate Interlock Factor AGG :

$N=16$; $R^2=0.988$; $COV=2.45\%$. Limits: $5 \leq (AGG/kl)$; $0.05 \leq (a/l) \leq 0.3$

$$\frac{\sigma_{AGG}}{\sigma_{West}} = 0.99864 - 0.51237(a/l) - 0.0762 \ln(AGG/kl) + 0.00315 \ln^2(AGG/kl) + 0.015936(a/l)^2 \ln^2(AGG/kl)$$

Alternate Formula Used for $(a/l) > 5$:

$$\frac{\sigma_{AGG}}{\sigma_{West}} = 1.04284 - 0.84692(a/l) - 0.09299 \ln(AGG/kl) + 0.06837(a/l) \ln(AGG/kl) + 0.63417(a/l)^2 + 0.0042 \ln^2(AGG/kl) - 0.000629(a/l) \ln(AGG/kl)^3$$

an adjustment introduced by the *PCA* so that they apply for $E_c = 4$ Mpsi [28 GPa]. This modulus was considered more typical of current construction practices. Values of σ_{eq} were then obtained using these regression equations as follows:

$$\sigma_{eq} = \frac{6 M_e}{h^2} * f_1 * f_2 * f_3 * f_4 \quad (7)$$

where f_i are additional adjustment factors (see Eq. 8).

Adjustment factor f_1 is related to the *PCA* assumption that the area of contact under a 6-kip [27 kN] wheel is 70 in.² [452 cm²]. Each wheel in a "standard" 18-kip [80 kN] single axle (SA) or 36-kip [161 kN] tandem (TA) carries only 4.5 kips [20 kN], and therefore, a slightly smaller contact area may be expected. Factor f_1 accounts for the effect of this difference in contact areas on *J-SLAB* stresses.

Table 2. PCA Stress Prediction (Courtesy of PCA)**(a) J-SLAB inputs for PCA Database**

Slab characteristics: $h=4, 6, 8, 10$ and 12 in.; $E_c = 5$ Mpsi [6 Mpsi];
 $\mu = 0.15$; $L = 180$ in.

Subgrade modulus: $k = 100, 300, 500$ psi/in.

Load configuration: $p = 64.29$ psi [130.5 psi]; Contact Area = 7 by 10 in. = 70 in.²; 18-kip SA: Dual tires, spaced at $S = 12$ in.; 36-kip TA: As for SA, with tandem axle spacing $t = 50$ in. [$t = 51.2$ in.]; Both SA and TA: Axle width (distance between centers of duals), $D = 72$ in. [$D = 90$ in.]

Concrete Shoulder: Assume Aggregate Interlock, $AGG = 25,000$ psi

Note: Figures in square brackets, [], pertain to Danish conditions assumed in this investigation (Courtesy of Aalborg Portland).

(b) PCA Regression Equations for Max. Edge Moment, M_e (lb-in.)

No shoulder, Single Axle Load:

$$M_e = EQ1N = -1600 + 2525 \log l + 24.42 l + 0.204 l^2$$

No shoulder, Tandem Axle Load:

$$M_e = EQ5N = 3029 - 2966.8 \log l + 133.69 l - 0.0632 l^2$$

Concrete shoulder, Single Axle Load:

$$M_e = EQ2N = (-970.4 + 1202.6 \log l + 53.587 l)(0.8742 + 0.01088 k^{0.447})$$

Concrete shoulder, Tandem Axle Load:

$$M_e = EQ6N = (2005.4 - 1980.9 \log l + 99.008 l)(0.8742 + 0.01088 k^{0.447})$$

$$f_1 = \left[\frac{18}{24} \right]^{0.94} * \left[\frac{24}{18} \right] = 1.0174 \quad \text{for single axles}$$

$$f_1 = \left[\frac{36}{48} \right]^{0.94} * \left[\frac{48}{36} \right] = 1.0174 \quad \text{for tandem axles}$$

$$f_2 = 0.892 + \left[\frac{h}{85.71} \right] - \left[\frac{h^2}{3000} \right] \approx 0.967 \quad \{h: \text{in inches}\} \quad (8)$$

$$f_3 = 0.894$$

$$f_4 = \frac{1}{1.235 * 0.85} = 0.952607$$

Adjustment factor f_2 is a stress reduction factor accounting for the support provided by the subgrade extending beyond the slab edges. This contribution is ignored by the conventional dense liquid (Winkler) idealization employed in *J-SLAB*. *PCA* recommended values for f_2 based on results from computer program *MATS*, developed for analysis and design of mat foundations, combined footings and slabs-on-grade (ref. 24). Note that these values of f_2 are used only for the "No Concrete Shoulder" (NS) cases. When a concrete shoulder is present (WS cases), f_2 is set to 1.0.

Adjustment factor f_3 accounts for the effect of truck placement on the edge stress, as determined from truck encroachment data and fatigue considerations. The value of $f_3 = 0.894$ used in the *1984 PCA Guide* reflects "the most severe condition" of 6% truck encroachment. Finally, adjustment factor f_4 accounts for the increase in concrete strength with age after the 28th day, along with a reduction in concrete strength (due to material variability) by one coefficient of variation (COV), assumed to be 15%.

The combined effect of the four adjustment factors is to reduce the moment, M_e , calculated using the regression equations in Table 2b. This reduction has a mean value of 0.83 (range: 0.80 to 0.85) for the NS cases, and is constant at 0.86 for the WS cases.

Tables 3 and 4 show the calculations performed for the SA/NS and TA/NS cases, respectively. Similarly, Tables 5 and 6 present computations for the corresponding SA/WS and TA/WS cases. Comparison of Columns H and I in these Tables confirms that the stresses calculated in this manner are, indeed, the "Equivalent Stresses" given in Tables 6a and 6b of the *1984 PCA Guide*.

5 REPRODUCTION OF PCA STRESSES USING DIMENSIONAL ANALYSIS

A small personal computer spreadsheet has been prepared to perform the calculations required by the proposed stress prediction algorithm. These computations are included in Tables 3 through 6, beginning with Column K in each case. The last column in these Tables presents a comparison of results obtained using the dimensional analysis (DA) approach with those determined using the *PCA* regression formulae. The effect of the four *PCA* adjustment factors, f_i , may be ignored in such a comparison. It is observed that the proposed DA methodology reproduces the *PCA* stresses with adequate precision. The mean values of the ratio (DA:PCA) range from 0.92 to 1.01, with an average COV of 4.5%. The largest discrepancies are observed in the stresses pertaining to tandem axles, with or without a concrete shoulder. A finite element mesh that is adequate for the analysis of a SA may be expected to lead to less precise stresses when a TA is applied, in view of the multiplicity of tire prints in the latter. Furthermore, a_{eq} for a TA is often larger than the radii envisioned by Westergaard in the derivation of his equations (refs 25, 20). In applying Westergaard's edge stress equation to cases involving a load size ratio, (a/l) , in excess of 0.5, it was considered preferable to use the two formulae termed "alternate" in Table 1. The first of these is a direct modification of the Westergaard equation and was derived on the basis of several runs with high (a/l) ratios using computer program *H-51* (ref. 26). The other is a replacement load transfer formula developed using additional *ILLI-SLAB* results pertaining to larger (a/l) values.

6 EFFECT OF "SUPER-SINGLES"

With the proposed stress prediction algorithm, stresses can be recalculated with great ease and with adequate precision any time the user desires to change one or more of the input parameters involved. Conventional prediction models based exclusively on statistics are not as reliable when data other than those employed in their derivation are examined. This observation addresses the heart of the perpetual "inadequate database" argument voiced in data interpretation studies, including those pertaining to the AASHO Road Test (ref. 22), as well as to the more extensive SHRP experiment (ref. 27).

As an illustration of these comments, the personal computer spreadsheet prepared in this investigation was used to assess the effects of introducing a "super-single" tire instead of a set of duals, at each end of a conventional single or tandem truck axle. Columns B

Table 3. Calculations for Single Axle, No Shoulder (SA/NS)

E = 4 Mpsi			p = 64.29 psi		S/a = 2.5422			f1 = 1.017												
mu = 0.15			a = 4.7201 in.		D = 72 in.			f3 = 0.894												
L = 180 in.			S = 12 in.		t = 50 in.			f4 = 0.952												
			EQ1N 6M/h2		Red. Guid P=9k			P=9k			P=9k			P=9k						
			Regr. Regr.		f1*f2			Regr. Eqvt PCA			corr.			D/l corr.			L/l corr. T/J			
h	k	l	Mom.	Str.	f2	*f3*f4	Eq.st.	Str.	Regr.	a/l	aeq/a	aeq/l	Wstgd	D/l	fact.	Wstgd	L/l	Fact.	Wstgd	Ratio
in.	pci	in.	lb-in.	psi			psi	psi	sh2/P				sh2/P			sh2/P			sh2/P	
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
4	100	21.6	2393.3	897.4	0.93	0.80	725.8	726	1.59	0.21	1.62	0.35	1.60	3.33	1	1.60	8.32	1	1.60	1.00
6	100	29.2	2994.1	499.0	0.95	0.82	410.7	411	1.99	0.16	1.63	0.26	1.97	2.45	1.04	2.06	6.14	1	2.06	1.03
8	100	36.3	3497.5	327.8	0.96	0.83	273.8	274	2.33	0.12	1.64	0.21	2.24	1.98	1.06	2.38	4.95	0.99	2.38	1.02
10	100	42.9	3949.9	236.9	0.97	0.84	200.2	200	2.63	0.10	1.64	0.18	2.45	1.67	1.08	2.66	4.18	0.98	2.61	0.99
12	100	49.2	4372.1	182.1	0.98	0.85	155.3	155	2.91	0.09	1.64	0.15	2.63	1.46	1.10	2.90	3.65	0.96	2.78	0.95
4	300	16.4	1925.0	721.9	0.93	0.80	583.8	584	1.28	0.28	1.61	0.46	1.29	4.38	1	1.29	10.9	1	1.29	1.00
6	300	22.2	2447.1	407.8	0.95	0.82	335.7	336	1.63	0.21	1.62	0.34	1.63	3.23	1	1.63	8.08	1	1.63	1.00
8	300	27.6	2869.2	268.9	0.96	0.83	224.6	225	1.91	0.17	1.63	0.27	1.89	2.60	1.04	1.98	6.51	1	1.98	1.03
10	300	32.6	3237.4	194.2	0.97	0.84	164.1	164	2.15	0.14	1.64	0.23	2.10	2.20	1.05	2.22	5.51	1	2.22	1.03
12	300	37.4	3572.6	148.8	0.98	0.85	126.9	127	2.38	0.12	1.64	0.20	2.27	1.92	1.06	2.43	4.80	0.99	2.42	1.01
4	500	14.4	1724.5	646.7	0.93	0.80	522.9	523	1.14	0.32	1.60	0.52	1.15	4.98	1	1.15	12.4	1	1.15	1.00
6	500	19.5	2219.1	369.8	0.95	0.82	304.4	304	1.47	0.24	1.62	0.39	1.49	3.67	1	1.49	9.18	1	1.49	1.00
8	500	24.3	2613.2	244.9	0.96	0.83	204.6	205	1.74	0.19	1.63	0.31	1.74	2.96	1.04	1.81	7.40	1	1.81	1.04
10	500	28.7	2952.8	177.1	0.97	0.84	149.7	150	1.96	0.16	1.63	0.26	1.94	2.50	1.04	2.04	6.26	1	2.04	1.03
12	500	32.9	3258.5	135.7	0.98	0.85	115.7	115	2.17	0.14	1.64	0.23	2.11	2.18	1.05	2.23	5.46	1	2.23	1.03
			avg		0.96	0.83			1.95	0.18		0.29		2.77	1.04			0.99	1.98	1.01
			min		0.93	0.80			1.14	0.09		0.15		1.46	1			0.96	1.15	0.95
			max		0.98	0.85			2.91	0.32		0.52		4.98	1.10			1	2.78	1.04
			cov		0.01	0.01			0.24	0.34		0.33		0.34	0.03			0.01	0.23	0.02

Table 4. Calculations for Tandem Axle, No Shoulder (TA/NS)

			EQ5N 6M/h2		Red. Guid P=18k			P=18k			P=18k			L/l P=18k						
			Regr. Regr.		f1*f2			Regr. Eqvt PCA			aeqs			t/aeq/aeq corr.			D/l corr. Corr. corr. T/J			
h	k	l	Mom.	Str.	f2	*f3*f4	Eq.st.	Str.	Regr.	/a	t/l	aeqs	aeq/aeq	/l	Wstgd	fact.	Wstgd	Fact.	Wstgd	Ratio
in.	pci	in.	lb-in.	psi			psi	psi	sh2/P						sh2/P		sh2/P		sh2/P	
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
4	100	21.6	1929.1	723.4	0.93	0.80	585.0	585	0.64	1.62	2.31	6.50	2.53	0.90	0.62	1	0.62	1	0.62	0.97
6	100	29.2	2539.5	423.2	0.95	0.82	348.4	348	0.84	1.63	1.70	6.46	2.65	0.70	0.85	1.04	0.89	1	0.89	1.06
8	100	36.3	3175.4	297.6	0.96	0.83	248.6	249	1.05	1.64	1.37	6.44	2.75	0.58	1.03	1.06	1.10	1.00	1.10	1.04
10	100	42.9	3811.9	228.7	0.97	0.84	193.2	193	1.27	1.64	1.16	6.43	2.83	0.51	1.17	1.08	1.27	0.95	1.22	0.96
12	100	49.2	4440.8	185.0	0.98	0.85	157.7	158	1.48	1.64	1.01	6.42	2.91	0.46	1.30	1.10	1.43	0.91	1.31	0.89
4	300	16.4	1601.5	600.5	0.93	0.80	485.6	486	0.53	1.61	3.04	6.56	2.42	1.12	0.45	1	0.45	1	0.45	0.84
6	300	22.2	1975.7	329.2	0.95	0.82	271.0	271	0.65	1.62	2.24	6.49	2.54	0.87	0.64	1	0.64	1	0.64	0.98
8	300	27.6	2397.5	224.7	0.96	0.83	187.7	188	0.79	1.63	1.81	6.47	2.63	0.73	0.80	1.04	0.84	1	0.84	1.05
10	300	32.6	2835.4	170.1	0.97	0.84	143.7	144	0.94	1.64	1.53	6.45	2.70	0.64	0.94	1.05	0.99	1	0.99	1.05
12	300	37.4	3277.6	136.5	0.98	0.85	116.4	116	1.09	1.64	1.33	6.44	2.76	0.57	1.05	1.06	1.13	0.99	1.12	1.02
4	500	14.4	1506.7	565.0	0.93	0.80	456.9	457	0.50	1.60	3.45	6.60	2.38	1.24	0.38	1	0.38	1	0.38	0.76
6	500	19.5	1790.6	298.4	0.95	0.82	245.6	246	0.59	1.62	2.55	6.52	2.49	0.97	0.56	1	0.56	1	0.56	0.94
8	500	24.3	2130.2	199.7	0.96	0.83	166.8	167	0.71	1.63	2.05	6.48	2.57	0.81	0.71	1.04	0.74	1	0.74	1.04
10	500	28.7	2491.7	149.5	0.97	0.84	126.3	126	0.83	1.63	1.73	6.46	2.64	0.71	0.83	1.04	0.88	1	0.88	1.06
12	500	32.9	2862.1	119.2	0.98	0.85	101.6	102	0.95	1.64	1.51	6.45	2.70	0.63	0.95	1.05	1.00	1	1.00	1.05
			avg		0.83				0.86		1.92	6.48	2.63	0.76		1.04		0.99	0.85	0.98
			min		0.80				0.50		1.01	6.42	2.38	0.46		1		0.91	0.38	0.76
			max		0.85				1.48		3.45	6.60	2.91	1.24		1.10		1.00	1.31	1.06
			cov		0.01				0.31		0.34	0.00	0.05	0.28		0.03		0.02	0.31	0.08

Table 5. Calculations for Single Axle, With Shoulder (SA/WS)

E= 4 Mpsi			p= 64.29 psi		f1= 1.017												
mu=0.15			a= 4.7201 in.		f3= 0.894												
L= 180 in.			AGG= 25000 lb/in2		f4= 0.952												
			EQ2N 6M/h2		Red. Guid P=9k P=9k						AGG P=9k						
			Regr. Regr.		f1*f2		Regr. Eqvt PCA corr.				AGG/ Corr. corr.		O/I				
h	k	l	Mom.	Str.	f2	*f3*f4	Eq.st.	Str.	Regr.	Wstgd	aeq/l	kl	Fact.	Wstgd	Ratio		
in.	pci	in.	lb-in.	psi			psi	psi	sh2/P	sh2/P				sh2/P			
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P		
4	100	21.6	1720.2	645.0	1	0.86	558.9	559	1.14	1.60	0.35	11.5	0.66	1.06	0.92		
6	100	29.2	2267.5	377.9	1	0.86	327.4	327	1.51	2.06	0.26	8.53	0.71	1.48	0.98		
8	100	36.3	2738.3	256.7	1	0.86	222.4	222	1.82	2.38	0.21	6.87	0.75	1.80	0.98		
10	100	42.9	3162.7	189.7	1	0.86	164.4	164	2.10	2.61	0.18	5.81	0.78	2.04	0.97		
12	100	49.2	3554.9	148.1	1	0.86	128.3	128	2.36	2.78	0.15	5.07	0.80	2.23	0.94		
4	300	16.4	1389.8	521.1	1	0.86	451.5	452	0.92	1.29	0.46	5.07	0.65	0.84	0.91		
6	300	22.2	1867.8	311.3	1	0.86	269.7	270	1.24	1.63	0.34	3.74	0.72	1.19	0.96		
8	300	27.6	2273.1	213.1	1	0.86	184.6	185	1.51	1.98	0.27	3.01	0.77	1.54	1.01		
10	300	32.6	2635.0	158.1	1	0.86	136.9	137	1.75	2.22	0.23	2.55	0.80	1.80	1.02		
12	300	37.4	2967.2	123.6	1	0.86	107.1	107	1.97	2.42	0.20	2.22	0.83	2.02	1.02		
4	500	14.4	1258.1	471.8	1	0.86	408.8	409	0.83	1.15	0.52	3.45	0.70	0.81	0.97		
6	500	19.5	1713.6	285.6	1	0.86	247.4	247	1.14	1.49	0.39	2.55	0.73	1.09	0.95		
8	500	24.3	2097.1	196.6	1	0.86	170.3	170	1.39	1.81	0.31	2.05	0.78	1.42	1.01		
10	500	28.7	2437.8	146.2	1	0.86	126.7	127	1.62	2.04	0.26	1.73	0.81	1.67	1.03		
12	500	32.9	2749.5	114.5	1	0.86	99.26	99	1.83	2.23	0.23	1.51	0.84	1.89	1.03		
								avg	1.54		0.29	4.38	1.10	1.53	0.98		
								min	0.83		0.15	1.51	0.65	0.81	0.91		
								max	2.36		0.52	11.5	2.23	2.23	1.03		
								cov	0.27		0.33	0.62	0.38	0.28	0.03		

Table 6. Calculations for Tandem Axle, With Shoulder (TA/WS)

			EQ6N 6M/h2		f1*f2		Red. Guid P=18k P=18k				AGG P=18k						
			Regr. Regr.		*f3*f4		Regr. Eqvt PCA corr.		aeqt AGG/ Corr. corr.		O/I						
h	k	l	Mom.	Str.	f2		Eq.st.	str.	Regr.	Wstgd	/l	kl	Fact.	Wstgd	Ratio		
in.	pci	in.	lb-in.	psi			psi	psi	sh2/P	sh2/P				sh2/P			
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P		
4	100	21.6	1440.4	540.1	1	0.86	468.0	468	0.48	0.62	0.90	11.5	0.73	0.46	0.96		
6	100	29.2	1919.1	319.8	1	0.86	277.1	277	0.63	0.89	0.70	8.53	0.68	0.61	0.95		
8	100	36.3	2411.2	226.0	1	0.86	195.8	196	0.80	1.10	0.58	6.87	0.67	0.74	0.92		
10	100	42.9	2902.1	174.1	1	0.86	150.8	151	0.96	1.22	0.51	5.81	0.68	0.83	0.86		
12	100	49.2	3387.4	141.1	1	0.86	122.2	122	1.12	1.31	0.46	5.07	0.65	0.86	0.76		
4	300	16.4	1240.2	465.0	1	0.86	402.9	403	0.41	0.45	1.12	5.07	0.87	0.39	0.95		
6	300	22.2	1560.7	260.1	1	0.86	225.3	225	0.52	0.64	0.87	3.74	0.75	0.48	0.93		
8	300	27.6	1910.5	179.1	1	0.86	155.1	155	0.63	0.84	0.73	3.01	0.72	0.61	0.95		
10	300	32.6	2269.5	136.1	1	0.86	117.9	118	0.75	0.99	0.64	2.55	0.71	0.71	0.94		
12	300	37.4	2630.3	109.5	1	0.86	94.96	95	0.87	1.12	0.57	2.22	0.72	0.81	0.92		
4	500	14.4	1194.6	448.0	1	0.86	388.1	388	0.39	0.38	1.24	3.45	0.96	0.37	0.93		
6	500	19.5	1453.8	242.3	1	0.86	209.9	210	0.48	0.56	0.97	2.55	0.79	0.44	0.92		
8	500	24.3	1749.1	163.9	1	0.86	142.0	142	0.58	0.74	0.81	2.05	0.74	0.55	0.95		
10	500	28.7	2058.1	123.4	1	0.86	106.9	107	0.68	0.88	0.71	1.73	0.73	0.65	0.94		
12	500	32.9	2372.1	98.83	1	0.86	85.63	86	0.79	1.00	0.63	1.51	0.74	0.74	0.94		
								avg	0.67		0.76	4.38	0.74	0.62	0.92		
								min	0.39		0.46	1.51	0.65	0.37	0.76		
								max	1.12		1.24	11.5	0.96	0.86	0.96		
								cov	0.30		0.28	0.62	0.10	0.25	0.05		

Table 7. Comparison of Normalized Edge Bending Stress
a) Single Axle Load (SA/NS and SA/WS)

SA/NS EUR.					SA/WS EUR.					
PCA	SSIN	SSIN	Ratio	Ratio	PCA	SSIN	SSIN	Ratio	Ratio	
DA	DA	DA	B/A	C/A	DA	DA	DA	B/A	C/A	
A	B	C	D	E	A	B	C	D	E	
1.60	1.77	2.35	1.10	1.46	1.06	1.21	1.74	1.14	1.64	
2.06	2.24	2.75	1.08	1.33	1.48	1.65	2.15	1.11	1.44	
2.38	2.56	3.08	1.07	1.29	1.80	1.98	2.49	1.09	1.38	
2.61	2.81	3.30	1.07	1.26	2.04	2.23	2.74	1.09	1.34	
2.78	2.99	3.47	1.07	1.24	2.23	2.43	2.94	1.08	1.31	
1.29	1.44	2.00	1.11	1.54	0.84	0.98	1.52	1.16	1.79	
1.63	1.81	2.38	1.10	1.45	1.19	1.36	1.94	1.14	1.62	
1.98	2.15	2.66	1.08	1.34	1.54	1.71	2.25	1.11	1.46	
2.22	2.40	2.91	1.08	1.31	1.80	1.98	2.53	1.10	1.40	
2.42	2.61	3.12	1.07	1.28	2.02	2.21	2.77	1.09	1.37	
1.15	1.29	1.84	1.12	1.59	0.81	0.87	1.41	1.07	1.72	
1.49	1.65	2.22	1.11	1.49	1.09	1.25	1.83	1.15	1.68	
1.81	1.98	2.50	1.08	1.37	1.42	1.59	2.16	1.11	1.51	
2.04	2.21	2.72	1.08	1.33	1.67	1.85	2.42	1.10	1.44	
2.23	2.42	2.93	1.08	1.30	1.89	2.09	2.66	1.10	1.40	
1.98	2.16	2.68	1.09	1.37	avg	1.53	1.69	2.24	1.11	1.50
1.15	1.29	1.84	1.07	1.24	min	0.81	0.87	1.41	1.07	1.31
2.78	2.99	3.47	1.12	1.59	max	2.23	2.43	2.94	1.16	1.79
0.23	0.22	0.16	0.01	0.07	cov	0.28	0.27	0.20	0.02	0.09

b) Tandem Axle Load (TA/NS and TA/WS)

TA/NS EUR.					TA/WS EUR.					
PCA	SSIN	SSIN	Ratio	Ratio	PCA	SSIN	SSIN	Ratio	Ratio	
DA	DA	DA	B/A	C/A	DA	DA	DA	B/A	C/A	
A	B	C	D	E	A	B	C	D	E	
0.62	0.72	1.01	1.15	1.62	0.46	0.50	0.67	1.09	1.46	
0.89	0.98	1.22	1.10	1.36	0.61	0.66	0.82	1.08	1.34	
1.10	1.18	1.37	1.07	1.24	0.74	0.80	0.95	1.07	1.27	
1.22	1.30	1.46	1.06	1.19	0.83	0.83	1.00	0.99	1.20	
1.31	1.39	1.53	1.05	1.16	0.86	0.93	1.09	1.08	1.26	
0.45	0.54	0.85	1.20	1.89	0.39	0.43	0.60	1.09	1.51	
0.64	0.74	1.03	1.15	1.60	0.48	0.54	0.73	1.10	1.50	
0.84	0.93	1.17	1.10	1.39	0.61	0.66	0.84	1.09	1.39	
0.99	1.08	1.30	1.08	1.31	0.71	0.78	0.96	1.08	1.35	
1.12	1.20	1.38	1.07	1.23	0.81	0.88	1.04	1.08	1.28	
0.38	0.46	0.78	1.22	2.05	0.37	0.40	0.56	1.09	1.53	
0.56	0.65	0.95	1.17	1.70	0.44	0.49	0.69	1.11	1.54	
0.74	0.83	1.09	1.12	1.47	0.55	0.61	0.80	1.10	1.44	
0.88	0.97	1.20	1.10	1.37	0.65	0.71	0.90	1.09	1.39	
1.00	1.09	1.31	1.08	1.30	0.74	0.81	1.01	1.09	1.35	
0.85	0.94	1.18	1.12	1.46	avg	0.62	0.67	0.84	1.08	1.39
0.38	0.46	0.78	1.05	1.16	min	0.37	0.40	0.56	0.99	1.20
1.31	1.39	1.53	1.22	2.05	max	0.86	0.93	1.09	1.11	1.54
0.31	0.28	0.18	0.04	0.17	cov	0.25	0.24	0.18	0.02	0.07

and D of Table 7 show that for the input parameters assumed by the *PCA* (most notably the same tire print area and pressure), "super-singles" cause approximately a 10% increase in calculated stresses. When input parameters reflecting current European (Danish) practice are employed (see Table 2), an additional 35% increase in calculated maximum edge stresses is observed (Columns C and E). The bulk of this highly detrimental increase in stress is due to the

higher European contact pressure, and the concomitant reduction in the radius of the applied load. This finding sheds new light on the effect of contact pressure on concrete pavements, sometimes considered to be insignificant (ref. 28). Application of a temperature differential, although not considered here, can magnify this load radius effect further (ref. 6).

Parametric studies are greatly facilitated using the DA approach. The effect of some of the input parameters is illustrated in Table 8, for the typical case of a 12-in. [30-cm] slab resting on a subgrade whose *k*-value is 300 psi/in. [81 MN/m³].

7 CONCLUSION

Westergaard's equations may be employed in the analysis of complex loading conditions provided that appropriate "correction factors" are established every time one of Westergaard's limiting assumptions must be eliminated (ref. 2). A set of such factors have been derived by Salsilli (ref. 17) using a novel combination of dimensional analysis and statistical techniques in data interpretation. Formulae have been developed for the prediction of the critical edge stress arising in a concrete slab-on-grade pavement. The database for this derivation consisted of a relatively small factorial of finite element runs using the *ILLI-SLAB* program.

The proposed edge stress predictive algorithm was implemented on a very efficient personal computer spreadsheet and was used in this study to reproduce with remarkable precision the "Equivalent Stresses" given in the widely used *1984 PCA Guide*. Subsequently, the dimensional analysis approach was applied in an examination of the increasingly popular "super-single" tires. It is shown that the bulk of the significant detrimental effect of these tires is due to the reduction in tire contact area because of the simultaneous increase in tire contact pressure (commonly assumed to be equal to the inflation pressure). This challenges simplistic generalizations claiming that tire pressure effects in concrete pavements are unimportant.

This Paper offers an illustration of the potential that dimensional analysis possesses, and the significant advances that can be achieved when statistical methods of interpretation are applied only after the possibilities of more mechanistic approaches have been exhausted.

8 METRIC CONVERSION FACTORS

1 in.	=	25.4 mm
1 lb	=	4.44822 N
1 psi	=	6.89476 kPa
1 psi/in.	=	0.27145 MN/m ³

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Table 8. Sensitivity Analysis

TYPE	h	k	E	p	D	L	AGG	t	oh2	σ	Ratio
	in.	pci	Mpsi	psi	in.	in.	psi	in.	/P	psi	
"Standard" Condition											
SA/NS	12	300	6	130.5	90	180	-	-	3.12	195	1.00
SA/NS	8	300	6	130.5	90	180	-	-	2.66	374	1.91
SA/NS	16	300	6	130.5	90	180	-	-	3.4	119	0.61
SA/NS	12	50	6	130.5	90	180	-	-	3.67	229	1.17
SA/NS	12	500	6	130.5	90	180	-	-	2.93	183	0.93
SA/NS	12	300	4	130.5	90	180	-	-	2.97	185	0.95
SA/NS	12	300	8	130.5	90	180	-	-	3.22	201	1.03
SA/NS	12	300	6	90.0	90	180	-	-	2.88	180	0.92
SA/NS	12	300	6	180.0	90	180	-	-	3.29	205	1.05
SA/NS	12	300	6	130.5	75	180	-	-	3.2	200	1.02
SA/NS	12	300	6	130.5	125	180	-	-	3.04	190	0.97
SA/NS	12	300	6	130.5	90	144	-	-	3.01	188	0.96
SA/NS	12	300	6	130.5	90	216	-	-	3.15	196	1.00
SA/WS	12	300	6	130.5	90	180	25000	-	2.77	173	0.88
SA/WS	12	300	6	130.5	90	180	15000	-	2.89	180	0.92
SA/WS	12	300	6	130.5	90	180	50000	-	2.62	163	0.83
TA/NS	12	300	6	130.5	90	180	-	51.2	1.38	172	0.88
TA/NS	12	300	6	130.5	90	180	-	40	1.48	185	0.94
TA/NS	12	300	6	130.5	90	180	-	60	1.34	167	0.85
TA/WS	12	300	6	130.5	90	180	25000	51.2	1.04	130	0.66
TA/WS	12	300	6	130.5	90	180	15000	51.2	1.09	136	0.69
TA/WS	12	300	6	130.5	90	180	50000	51.2	0.99	123	0.63

Guide. Useful information concerning pavement design inputs appropriate for Denmark as a typical European country was supplied by *Aalborg Portland*, Copenhagen, Denmark.

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