Study Of An Active Suspension For Improved Ride Quality And Reduced Dynamic Wheel Loads

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ABSTRACT

An articulated vehicle suspension comprising a parallel combination of energy restoring and dissipative elements and a feedback controlled force generator is analyzed using *Linear Quadratic Gaussian* (*LQG*) control technique. An ideal active suspension scheme is formulated to minimize a quadratic performance measure comprising ride quality, cargo safety, suspension and tire dynamic deflections, and power requirements. The presence of the passive components yields a fail-safe active suspension design that reduces the power requirements and enhances the reliability. The ride quality and the dynamic wheel load performance characteristics of the actively suspended vehicle are compared to those of a vehicle with an "optimum" passive suspension to demonstrate the performance potential of the ideal suspension scheme. The results of the study reveal that the fail-safe active suspensions can yield performance benefits associated with ride quality as well as tire dynamic forces.

INTRODUCTION

The health and safety risks associated with the prolonged exposure to high levels of articulated vehicle vibration have prompted a demand for enhancement of ride quality performance of the vehicles. The high levels of vehicular vibration have also raised many concerns related to cargo safety. Furthermore, the dynamic wheel loads transmitted to the pavement are directly related to the different modes of vibration of the vehicle. The vibration response characteristics and thus the ride quality, cargo safety and dynamic tire loads of the vehicle are directly influenced by the suspension system design. Design of a passive vehicle suspension, however, involves a careful compromise among the various conflicting design requirements posed by the ride quality, handling and directional control performance, suspension deflections and tire forces. In view of the inherent performance limitations of passive suspensions, a number of active suspension concepts have been proposed to realize a better compromise among different design requirements. Although various active suspension designs have been developed for cars, light trucks and railway vehicles [1-6], their general implementation has

been severely limited due to high cost, poor reliability and high complexity. The demand for improved working conditions, cargo safety, low dynamic pavement loads, handling and stability performance of heavy vehicles, however, has been growing steadily. This, coupled with availability of more economical and reliable hardware, has resulted in renewed interest in active suspensions as feasible alternative to passive suspensions. While, drivers of passenger cars can benefit greatly from active suspensions, the potential performance benefits of active suspensions can best be harnessed to enhance the driver- and road-friendliness of heavy vehicles. Heavy trucks and articulated vehicles exhibit complex ride behavior and significant dynamic tire forces when compared to automobiles. A review of measured ride vibration data at the driver's seat revealed that the ride vibration levels of heavy trucks are 9 to 16 times higher than those of passenger cars [7]. Moreover, most heavy vehicle drivers are exposed to ride vibrations for 10 to 20 hours a day. In view of driver health and safety risks associated with prolonged exposure to high levels of ride vibrations, and significant dynamic tire forces resulting in accelerated pavement deterioration, the effectiveness of advanced suspensions need to be further investigated for use in heavy vehicles. In this paper, an ideal active suspension scheme based on full-state feedback is investigated to evaluate its performance potentials for freight vehicles. While the design aspects are briefly addressed, the performance characteristics of the ideal active suspension are compared to those of an "optimum" passive suspension to determine their potential performance benefits.

MATHEMATICAL FORMULATION

An articulated freight vehicle is analytically modeled as illustrated in Figure 1. The vehicle model comprises the tractor and semitrailer sprung masses each with bounce and pitch degrees of freedom (*DOF*). The tractor is represented by a planar rigid body, m_{s1}, supported on three axle suspensions. The semitrailer is modeled by a planar rigid body m_{s2}, and is assumed to have two independent axles. The drive and semitrailer axles are represented by independent axle

suspensions. The lumped unsprung mass of each wheel and axle assembly is represented by m_{ui} with only vertical *DOF*. The elastic property of each tire is characterized by a linear stiffness element, K_{ti}, assuming point contact with the road, and negligible wheel hop motion and damping properties. The suspension stiffness and damping, and tire stiffness coefficients are assumed to be linear. Each axle suspension is represented by a linear spring and a viscous damper in parallel with an active force generators. The control forces u_i ($i=1,...,5$) generated using a control scheme and the feedback from road excitations and vehicle response variables to reduce a given cost function or performance index. An active suspension thus hereafter refers to a parallel combination of passive elements and an active force generator. Table 1 describes the parameters of the five-axle articulated vehicle considered in this study.

To assess the performance potential benefits of the active suspension system, the vehicle model with passive suspension was initially analyzed to select the "optimum" passive suspension parameters. The passive suspension study resulted in the "optimum" suspension parameters listed in Table 2 [9].

Table 1. List of parameters for the baseline model [8].
(Based on half-a-vehicle)

| Description of the Parameter	Parameter Value
Tractor mass (m_{s1})	2,000.0 kg
Tractor pitch moment of inertia (I_{y1})	10,000.0 $kg.m^2$
Tractor front tire and axle assembly mass (m_{u1})	270.0 kg
Tractor leading/trailing rear axle assembly mass (m_{u2})	520.0 kg
Semitrailer mass (m_{s2})	14,000.0 kg
Semitrailer pitch moment of inertia (I_{y2})	300,000.0 $kg.m^2$
Semitrailer leading/trailing axle assembly mass (m_{u3})	340.0 kg
Tractor front tire stiffness (K_{t1})	788.0 kN/m
Tractor leading/trailing rear axle tire stiffness (K_{t2})	1,576.0 kN/m
Semitrailer leading/trailing axle tire stiffness (K_{t3})	1,576.0 kN/m
Fifth wheel stiffness (K_5)	20,000.0 kN/m
Fifth wheel damping coefficient (C_5)	200.0 $kN.s/m$
Distance from tractor CG to tractor steer axle (A_1)	1.5 m
Distance from tractor CG to tractor first drive axle (B_1)	2.8 m
Distance from tractor CG to second drive axle (B_2)	4.5 m
Distance from tractor CG to fifth wheel (B_5)	3.0 m
Distance from semitrailer CG to fifth wheel (A_2)	6.0 m
Distance from semitrailer CG to first semitrailer axle (B_3)	5.6 m
Distance from semitrailer CG to semitrailer second axle (B_4)	6.8 m
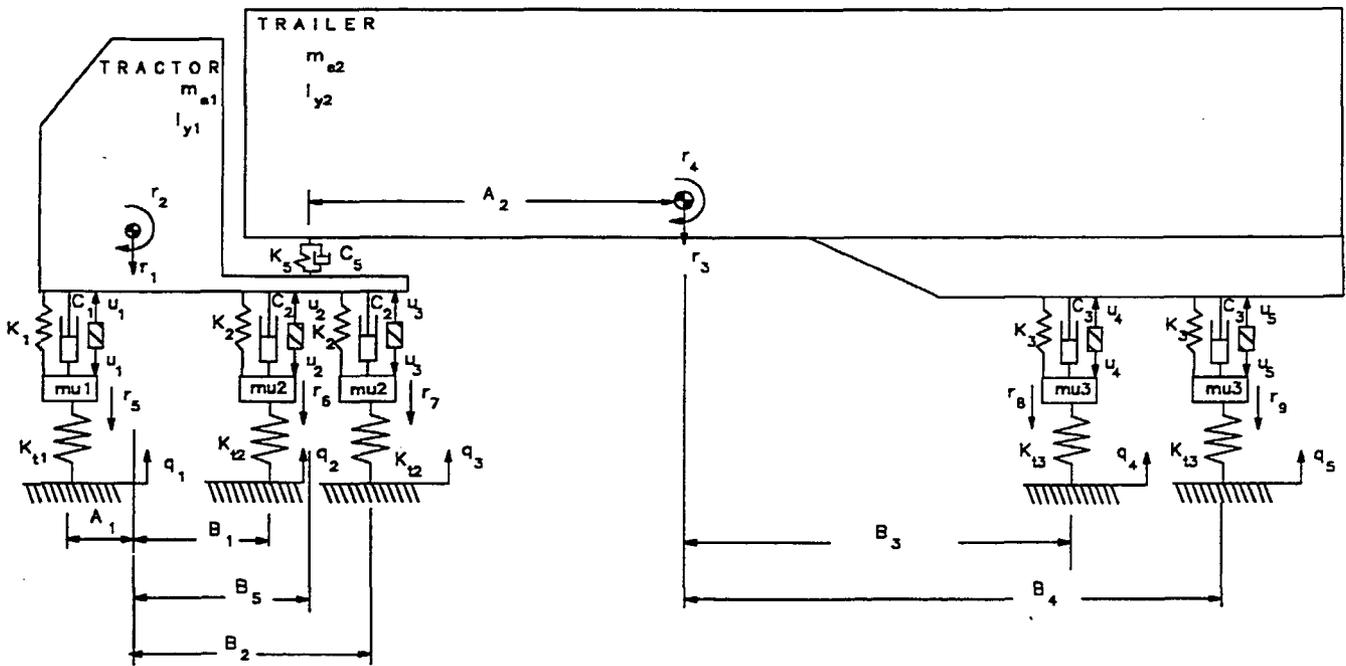
Figure 1. Tractor-semitrailer model

Table 2. "Optimum" passive suspension parameters [9]

	Stiffness (N/m)	Damping (Ns/m)
Suspension 1.	180 K	11 K
2.	300 K	22 K
3.	300 K	22 K
4.	300 K	33 K
5.	300 K	33 K

Assuming linear behavior for all elements and small angular displacements, a set of second order coupled differential equations characterizing the ride dynamics of the baseline actively suspended vehicle are derived and represented in the following matrix form;

$$M\ddot{r} + C\dot{r} + K_s r + Fu = K_T q \qquad (1)$$

where M, C and $K_s \in \mathbb{R}^{9\times9}$ are the mass, damping and stiffness matrices respectively. $F \in \mathbb{R}^{9\times5}$ is the matrix of control force coefficients, $K_T \in \mathbb{R}^{9\times5}$ is the tire stiffness matrix, $r \in \mathbb{R}^{9\times1}$ is the vector of generalized coordinates, $u \in \mathbb{R}^{5\times1}$ is the control forces vector, and $q \in \mathbb{R}^{5\times1}$ is the road disturbance vector.

Defining the new state vector y_y as;

$$y_y = \begin{Bmatrix} \dot{r} \\ r \end{Bmatrix}$$

The dynamics of the actively suspended tractor-semitrailer can be described by the first order matrix equation;

$$\dot{y}_y(t) = A_y y_y + B_y q + F_y u \qquad (2)$$

where;

$$A_y = \begin{bmatrix} -M^{-1}C & -M^{-1}K_s \\ I & 0 \end{bmatrix} , \ B_y = \begin{bmatrix} M^{-1}K_T \\ 0 \end{bmatrix} \ and \ F_y = \begin{bmatrix} -M^{-1}F \\ 0 \end{bmatrix} \qquad (3)$$

where $A_y \in \mathbb{R}^{18\times18}$, $B_y \in \mathbb{R}^{18\times5}$, and $F_y \in \mathbb{R}^{18\times5}$ are matrices of vehicle and road constants. $I \in \mathbb{R}^{9\times9}$, is an identity matrix and $y_y \in \mathbb{R}^{18\times1}$ is the state vector.

The design of an active suspension for the tractor-semitrailer is, in principle, an optimization problem involving various performance measures related to ride quality, cargo safety, pavement loads, etc. *LQG* optimal control technique can be conveniently implemented for the design and analysis of the proposed active suspension system. *LQG* optimal control theory, however, requires the disturbance acting on the system to be white noise [10]. Since this condition is not explicitly satisfied here, a plant augmentation is carried out by. The stationary *Gaussian* random excitation vector q is thus obtained by passing a white noise process with covariance

$$E\left[\xi(t)\xi(\tau)\right] = 2\alpha v\sigma^2 \delta(t-\tau) \text{ through a first order filter;}$$

$$\dot{q} = A_q q + B_q \xi \qquad (4)$$

where;

$$A_q = \begin{bmatrix} -\alpha v & 0 & 0 & 0 & 0 \\ \alpha v + 2/\Delta_2 & -2/\Delta_2 & 0 & 0 & 0 \\ \alpha v + 2/\Delta_3 & 0 & -2/\Delta_3 & 0 & 0 \\ \alpha v + 2/\Delta_4 & 0 & 0 & -2/\Delta_4 & 0 \\ \alpha v + 2/\Delta_5 & 0 & 0 & 0 & -2/\Delta_5 \end{bmatrix} \ and \ B_q = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

where α is a coefficient that depends on the roughness of road irregularities and σ^2 is the variance of road irregularities, and v is the vehicle speed. Δ_n is the time delay between the first and the nth wheel.

EQ (2) can be rearranged along with the road excitation EQ (4) in the following augmented state-space form;

$$\dot{x} = Ax + B_1\xi + B_2 u \qquad (5)$$

where;

$$x = \begin{bmatrix} y_y \\ q \end{bmatrix}, A = \begin{bmatrix} A_y & B_y \\ 0 & A_q \end{bmatrix}, \ B_1 = \begin{bmatrix} 0 \\ B_q \end{bmatrix} \ and \ B_2 = \begin{bmatrix} F_y \\ 0 \end{bmatrix} \qquad (6)$$

where x is the 23×1 augmented state vector given by;

$$x = [\dot{r}_1 \ \dot{r}_2 \cdots \dot{r}_9 \ r_1 \ r_2 \cdots r_9 \ q_1 \ q_2 \ q_3 \ q_4 \ q_5]'$$

and " ' " designates the transpose. $A \in \mathbb{R}^{23\times23}$ is the system dynamics matrix, $B_1 \in \mathbb{R}^{23\times1}$ is the excitation disturbance vector, and $B_2 \in \mathbb{R}^{23\times5}$ is the control force matrix. The plant augmentation results in a system that is disturbed by a white noise ξ, and hence is well suited for the *LQG* optimal control technique.

THE PERFORMANCE INDEX

The performance index, in general, is selected to reflect specific design objectives. These design objectives include: ride quality, cargo safety, rattle space requirements and dynamic tire forces. While the ride quality related to work station design is frequently assessed using subjective rankings, many objective methods have been developed to assess the ride quality related to noise and vibration environment. Since the suspension design primarily determines the whole-body vibration environment of the driver's compartment, the ride quality related to vehicle vibration is addressed in this study. A relative measure of vehicle ride quality can be conveniently derived from the vertical (z_1) and pitch (z_2) acceleration response of the tractor sprung mass. The cargo safety is primarily related to the vibration levels transmitted to the cargo, and the performance index related to cargo safety can thus be expressed in terms of vertical (z_3) and pitch (z_4) acceleration responses of the semitrailer sprung mass. The relative deflections across the axle suspensions $(z_5, z_6, z_7, z_8, z_9)$ determine the rattle space requirements for the suspension system. Similarly, the relative deflections across the tire elements $(z_{10}, z_{11}, z_{12}, z_{13}, z_{14})$ directly relate to the dynamic forces transmitted to the pavements. The performance index also encompasses the control forces $(u_1$ to $u_5)$, in addition to the above penalized variables, to minimize the power requirements of the suspension systems. The performance index, therefore, includes the square of the following response variables ;

$z_1 = $ Tractor bounce acceleration $(\ddot{r}_1)$,

$z_2 = $ Tractor pitch acceleration $(\ddot{r}_2)$,

$z_3 = $ Semitrailer bounce acceleration $(\ddot{r}_3)$,

$z_4 = $ Semitrailer pitch acceleration $(\ddot{r}_4)$,

z_5 to $z_9 = $ Suspension deflections,

z_{10} to $z_{14} = $ Tire deflections,

u_1 to $u_5 = $ Actuator control forces.

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The performance variables z_1 to z_{14} can be related to the components of the augmented state vector and the actuator forces in the following form;

$$z = C_1 x + D_{12} u \qquad (7)$$

where $C_1 \in \mathbb{R}^{14 \times 23}$, is the state-to-performance variables transformation matrix and $D_{12} \in \mathbb{R}^{14 \times 5}$, is the control forces-to performance variables transformation matrix.

The performance index can be written as the weighted mean square sum of the performance variables and the control forces;

$$J = \lim_{T \to \infty} E \left\{ \int_0^T \left(\sum_{i=1}^{14} \mu_i z_i^2 + \sum_{j=1}^{5} \rho_j u_j^2 \right) dt \right\} \qquad (8)$$

where μ_i's and ρ_j's are weighting factors associated with the penalized response variables and the penalized actuator forces respectively. EQ (8) can be rearranged in the following format;

$$J = \lim_{T \to \infty} E \left\{ \int_0^T [x' \quad u'] \begin{bmatrix} Q & N \\ N' & R \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} dt \right\} \qquad (9)$$

where;
$$R = \hat{R} + D_{12}' \hat{Q} D_{12}, \quad Q = C_1' \hat{Q} C_1 \text{ and } N = C_1' \hat{Q} D_{12} \qquad (10)$$

and;
$$\hat{Q} = diag\ [\mu_1, \mu_2, \dots \mu_{14}]\ ; \quad \mu_i > 0$$
$$\hat{R} = diag\ [\rho_1, \rho_2, \dots \rho_5]\ ; \quad \rho_i > 0 \qquad (11)$$

SOLUTION OF THE STANDARD *LQG* PROBLEM

Design of an optimal controller for the multivariable active suspension system primarily involves the determination of the optimal control force vector u and its relation to the state vector x in the presence of a white noise input ξ. The control force vector u is optimum in the sense that it minimizes a quadratic cost function. This problem is well suited for the *LQG* technique, which is a well developed method for state-space design of multivariable systems. The *LQG* problem is thus formulated to determine the optimal control force vector u for the multivariable system governed by EQ (5), while minimizing the quadratic performance index J described in EQ (9). This is a *LQG* optimization control problem leading to an optimal control which is a linear algebraic function of the state vector [10,11];

$$u = -Kx \qquad (12)$$

where the constant control gain matrix K is given by;

$$K = R^{-1}(N' + B_2'P) \qquad (13)$$

and P is a symmetric, positive-definite solution of the matrix *Riccati* equation;

$$P(A - B_2 R^{-1} N') + (A - B_2 R^{-1} N')'P + P(-B_2 R^{-1} B_2')P - (-Q + NR^{-1}N') = 0 \qquad (14)$$

The steady-state, mean square response of the actively suspended vehicle can, therefore, be described by the covariance matrix X of the state vector $x(t)$ which is a solution of the following *Lyapunov* equation [10];

$$(A - B_2 K)X + X(A - B_2 K)' + 2\alpha v \sigma^2 B_1 B_1' = 0 \qquad (15)$$

The effects of variations in the suspension parameters on each component of the performance index can be evaluated from;

$$J_{zi} = \mu_i \sum_{m=1}^{23} \sum_{n=1}^{23} [C_1(i,m) - (D_{12}K)(i,m)][C_1(i,n) - (D_{12}K)(i,n)] X(m,n)$$
$$i = 1, \dots, 14. \qquad (16)$$

The contribution of the actuator control forces to the performance index can be similarly expressed as;

$$J_{uj} = \rho_j \sum_{m=1}^{23} \sum_{n=1}^{23} [K(j,m)][K(j,n)] X(m,n)\ ; \quad j = 1, \dots, 5 \qquad (17)$$

SELECTION OF THE WEIGHTING FACTORS

The *LQG* technique reduces the problem of designing the optimum active suspension to the selection of appropriate weighting coefficients. The active suspension performance is strongly dependent on the selected weighting factors. The selection of weighting factors, however, is quite complex due to excessive variations in the magnitudes of different performance variables. The weighting factors may be initially selected from the performance indices of the "optimum" passive suspension. Recalling that each of the variants of the performance index J is given by;

$$J_{zi} = \mu_i E(z_i^2) \qquad (18)$$

An equal importance of all the components of the performance index in the design of the active suspension implies that;

$$J_{z1} \approx J_{z2} \approx \dots \approx J_{z14} \qquad (19)$$

The weighting coefficients may thus be selected from;

$$\mu_i \approx \frac{1}{E(z_i^2)} \quad i = 1, \dots, 14 \qquad (20)$$

The determination of the appropriate weighting coefficients thus necessitates a prior knowledge of the mean square response of the performance variables. The mean square values $E(z_i^2)$, however, can be initially estimated from the response characteristics of the "optimum" passive suspension system. An initial set of weighting coefficients obtained from the analysis of the "optimum" passive suspension and EQ (20) is listed in Table 3. The control gain matrix K is then evaluated from EQ (13) and EQ (14). EQ (16) is solved to determine all the variants of the performance index, which are presented in Table 3. The analysis also requires careful selection of the weighting factors for the control forces ρ_j. ρ_j must be selected to ensure that sufficient power is available for the control. A preliminary study

revealed that the choice of $\rho_i=10^{-14}$ ensures availability of sufficient power.

Table 3. Weighting coefficients and corresponding active suspension performance indices

i	μ_i	J_{zi}
1	2.87e+0	0.63
2	1.41e+1	0.60
3	5.37e+0	0.85
4	8.33e+2	0.42
5	8.79e+5	0.93
6	4.60e+5	0.57
7	2.81e+5	1.05
8	2.35e+5	0.59
9	2.03e+5	0.58
10	4.12e+5	0.59
11	7.26e+5	0.41
12	5.39e+5	1.68
13	6.75e+5	0.74
14	6.47e+5	0.99

FREQUENCY DOMAIN ANALYSIS

Since the road excitation is assumed to be stationary and *Gaussian*, the vector of system response variables, z, is also stationary and *Gaussian* [12-14]. The frequency response function matrix $H(j\omega)$, $\in C^{14\times1}$, can thus be derived from EQs (5), (7) and (12);

$$H = \left\{(C_1 - D_{12}K)(sI - A + B_2K)^{-1}B_1\right\} \qquad (21)$$

The power spectral density of the response variables can be derived from [15];

$$S_z(\omega) = H S_\xi H^* \qquad (22)$$

where * denotes the complex conjugate transpose, $S_z(\omega) \in C^{14\times14}$ is the matrix of *PSD* of the response variables, and S_ξ is the spectral density of the white noise input excitation given by;

$$S_\xi = \frac{2av\sigma^2}{\pi} \qquad (23)$$

The diagonal elements of $S_z(\omega)$ provide the spectral densities of the response variables z_i.

PERFORMANCE CHARACTERISTICS OF THE ACTIVE SUSPENSION

An active suspension refers to a parallel combination of passive elements (spring and damper) and a feedback controlled force generator. Although an active suspension is often designed without the passive elements, the presence of passive components yields a fail-safe active suspension design with reduced power requirement. The role of passive elements within a fail-safe arrangement thus needs to be assessed through a study of the influence of passive suspension parameters on the response characteristics. EQ (16) is solved using the weighting coefficients given in Table 3 to determine the influence of the passive suspension parameters on each component of the performance index. The damping and stiffness elements that are present in parallel with the control force actuators are varied and the new control gain matrix K is evaluated accordingly. The results are examined to select the passive elements to best enhance the performance of the active suspension. The influence of the passive parameters on each component of the performance index is presented and discussed in the following section.

INFLUENCE OF PASSIVE SUSPENSION PARAMETERS

Figure 2 presents the influence of varying the damping and stiffness properties of the passive elements on the tractor bounce acceleration response. Since the control force gain matrix K is determined to minimize the performance index, variations in the passive damping and stiffness do not affect the performance index, as shown in the figure. While only one of the penalized variables is shown here for conciseness, identical behavior is also observed for all the other 13 performance variables. The results clearly illustrate that passive elements do not affect the suspension performance, provided the actuator gains are appropriately selected to account for the presence of the passive elements.

Although optimal control forces are derived to minimize the performance index for varying properties of the passive elements, the actuator forces are strongly dependent upon the passive damping and stiffness properties. EQ (17) is solved to study the effect of varying the passive suspension parameters on the actuator forces and to minimize the power requirements of the active suspension system. The results show the existence of an ideal range of passive suspension stiffness and damping values to realize the active suspension with minimum actuator forces magnitude. While any passive element can be placed in parallel with the active actuators without effecting the performance as shown in Figure 2, Figures 3a to 3e indicate that these elements directly affect the actuators effort. Figures 3a to 3e illustrate the contour plots of the control forces as a function of the restoring and dissipative properties of the passive elements. The damping and stiffness values corresponding to the center-most curve will yield minimum control force requirements. The actuator control force requirement for tractor axle suspension tends to decrease when the passive suspension stiffness is increased. The actuator force requirements are strongly affected by the dissipative properties of the passive components. An increase in the suspension damping properties, in general, results in a large increase in the power demand for the tractor axle suspensions.

The damping and stiffness values of passive elements, derived for the "optimum" passive suspension are used to realize the fail-safe active suspension. Although the selected passive suspension components do not yield minimal power requirements, the power demand is quite close to the optimal.
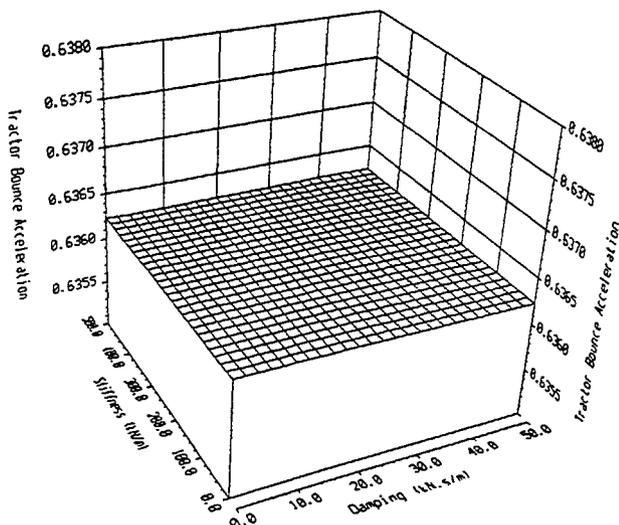
Figure 2. Weighted mean square acceleration of the tractor bounce for varying suspension stiffness and damping properties.
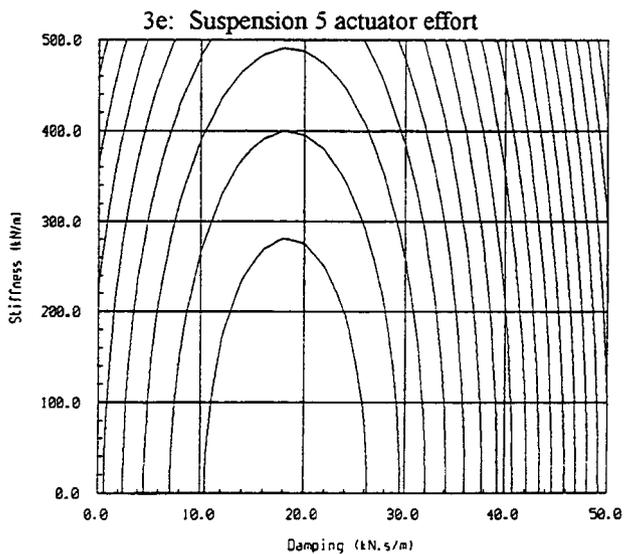

3a: Suspension 1 actuator effort

3b: Suspension 2 actuator effort

3c: Suspension 3 actuator effort

3d: Suspension 4 actuator effort
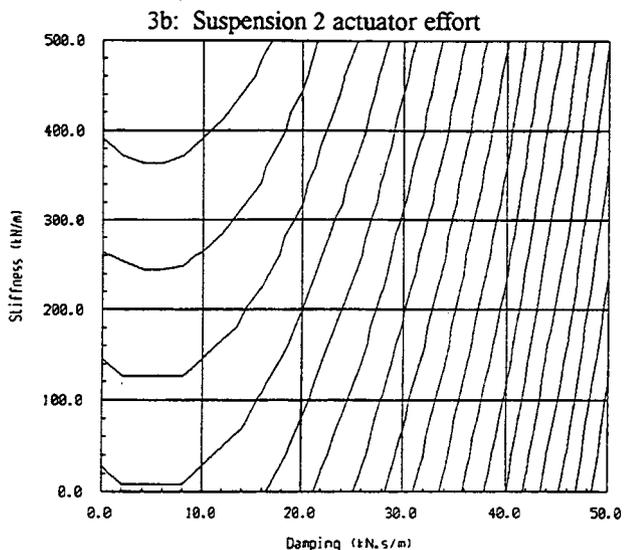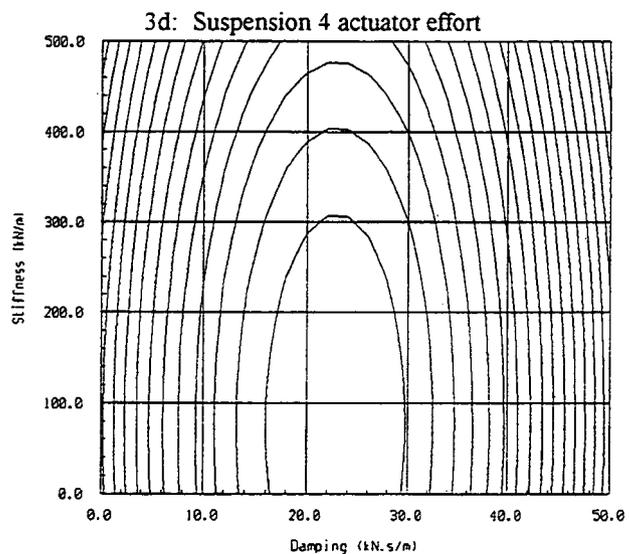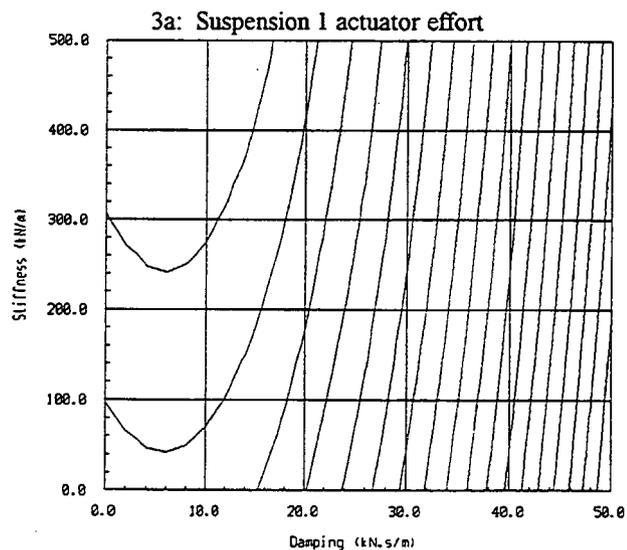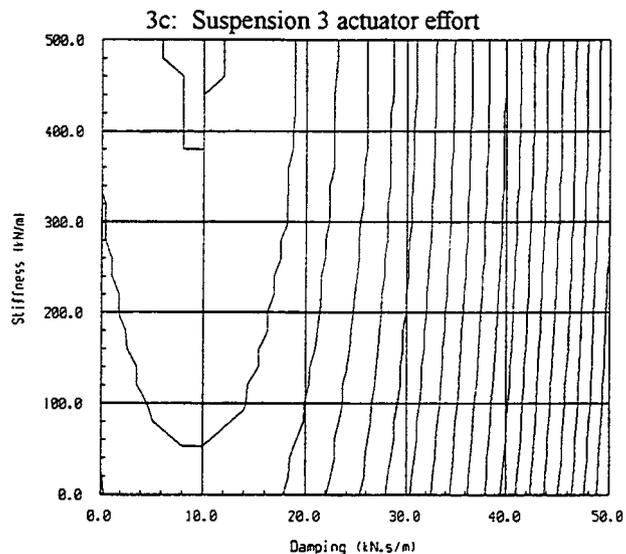

3e: Suspension 5 actuator effort

Figure 3. Iso-value curves of mean square actuator forces and the influence of passive suspension parameters

INFLUENCE OF VARIATIONS IN WEIGHTING COEFFICIENTS

The performance of an active suspension is strongly dependent on the choice of the weighting coefficients in the performance index J. The Weighting coefficients derived from the "optimum" passive suspension response listed in Table 3 and EQ (20), are initially used to determine the control gain matrix and the response characteristics of the vehicle model. The influence of variations in the weighting coefficients on the performance variables is then evaluated from EQ (16) and the weighting coefficients are selected to enhance the active suspension performance. The variations in the weighting coefficients are realized by introducing multiplication factors, Mt and Mf, with baseline coefficients listed in Table 3, and $\rho_i = 10^{+4}$.

The influence of weighting coefficients, μ_1 and μ_2, associated with tractor bounce and pitch acceleration response, on the components of the performance index is investigated to identify appropriate values of weighting coefficients. The coefficients μ_1 and μ_2 are varied simultaneously by varying the multiplication factor, Mt, in the range 1E-3 to 1E+2. The weighting factors, ρ_i ($i=1, ...,5$). are also varied by varying the corresponding multiplication factor, Mf, in the range 1E-20 to 1.00. The effect of varying these weighting coefficients on some of the performance variables is shown in Figures 4 to 7. An increase in μ_1 and μ_2 tends to emphasize the contribution of the tractor bounce and pitch acceleration response in the performance index J. The tractor bounce and pitch acceleration response thus decrease with increase in μ_1 and μ_2, as shown in Figures 4a and 4b. This, however, necessitates adequate weighting ρ_i to ensure availability of sufficient control force. The suspension performance approaches that of the passive suspension when the weighting factors ρ_i are increased due to insufficient control forces developed by the active force actuators. The influence of weighting coefficients on the suspension performance then becomes insignificant as reflected by the flat areas in the figures. An increase in μ_1 and μ_2 also results in improved semitrailer bounce acceleration response that may be attributed to the constraints posed by the articulation. However, increasing μ_1 and μ_2 beyond certain limit results in high semitrailer pitch acceleration, and suspensions and tires deflection, as shown in Figures 5b, 6 and 7. The expected values of these penalized variables, in extreme cases, exceed those derived for the passive suspension. A comprehensive parameteric study was carried out to study the influence of variations in other weighting coefficients on the performance index. While the results are not presented here, the following general observations were made: increasing a weighting coefficient, coupled with availability of sufficient actuating force, reduces the corresponding penalized variable and, generally, increases the other penalized variables.

Figures 4 to 7 clearly illustrate the strong dependency of the active suspension performance on the weighting coefficients, and the need to realize a compromise among the different components of the performance index. A set of weighting coefficients thus must be identified to realize an optimal suspension. It is desirable to design an active suspension to improve ride quality, cargo safety and reduce tires forces, while a reasonable increase in the suspension deflection can be considered acceptable. The weighting coefficients in the performance index are thus selected using an iterative procedure to attain the above design compromise. The procedure resulted in a number of weighting coefficients that satisfy the desired design compromise. A total of 5 sets of weighting coefficients are selected. The suspension schemes based on these weighting coefficients are designated as $AS1$ to $AS5$. The corresponding reduction or increase in the mean square value of the penalized variables relative to those derived for the "optimum" passive suspension [9] are listed in Table 4. All the five active suspension schemes, in general, yield improved performance when compared to that of the "optimum" passive suspension. Each of these schemes, however, accentuates certain particular aspects of the performance index. The suspension schemes $AS1$, $AS2$ and $AS3$ emphasize, to different extents, ride quality, cargo safety and tire forces. While the schemes $AS4$ and $AS5$ mostly emphasize the tire deflection response. The $AS2$ scheme yields best ride quality, while best cargo safety is achieved using $AS3$. A comparison of $AS4$ and $AS5$ schemes reveals that further improvements in the ride quality, cargo safety and tire deflection can be achieved by relaxing the suspension deflection requirements.

Table 4. Percent reduction(+)/increase(-) in the expected value of each of the penalized variables for 5 differently weighted active suspensions (compared to "optimum" passive)

| Performance variable	AS1 %	AS2 %	AS3 %	AS4 %	AS5 %
Tractor bounce acceleration	23.6	21.7	27.1	13.0	8.4
Tractor pitch acceleration	26.5	25.3	27.1	14.1	9.2
Semitrailer bounce acceleration	28.6	36.3	37.7	25.2	20.3
Semitrailer pitch acceleration	48.2	78.0	62.9	56.6	17.6
Suspension 1 Deflection	-6.9	2.0	0.1	-2.6	09.3
Suspension 2 Deflection	8.3	0.5	1.3	-7.5	13.4
Suspension 3 Deflection	1.18	3.6	3.6	-0.6	15.4
Suspension 4 Deflection	24.0	7.5	1.5	16.4	15.8
Suspension 5 Deflection	42.6	28.3	33.5	27.8	24.7
Tire 1 force	27.6	0.1	0.0	5.02	7.4
Tire 2 force	7.3	2.7	2.9	15.1	11.4
Tire 3 force	4.8	1.1	0.6	10.9	10.9
Tire 4 force	21.4	10.8	5.3	25.0	23.6
Tire 5 force	12.9	10.3	3.7	26.2	23.6
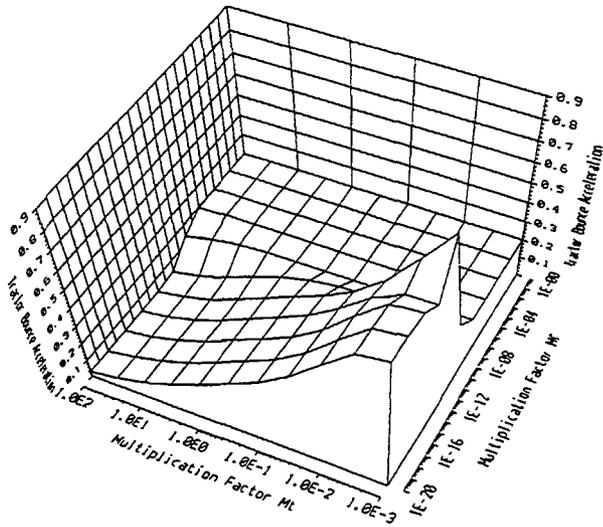
Figure 4a. Influence of variations in weighting coefficients on the mean square tractor bounce acceleration response
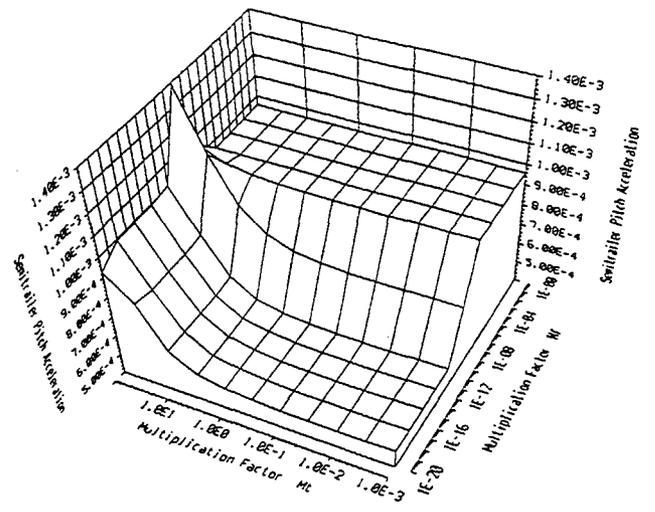
Figure 5b. Influence of variations in weighting coefficients on the mean square semitrailer pitch acceleration response
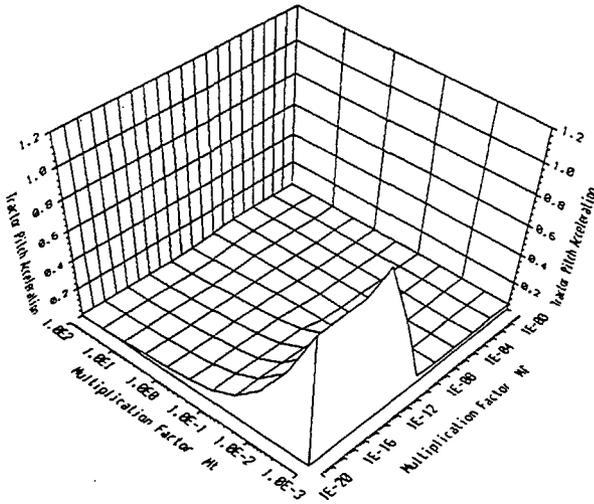
Figure 4b. Influence of variations in weighting coefficients on the mean square tractor pitch acceleration response
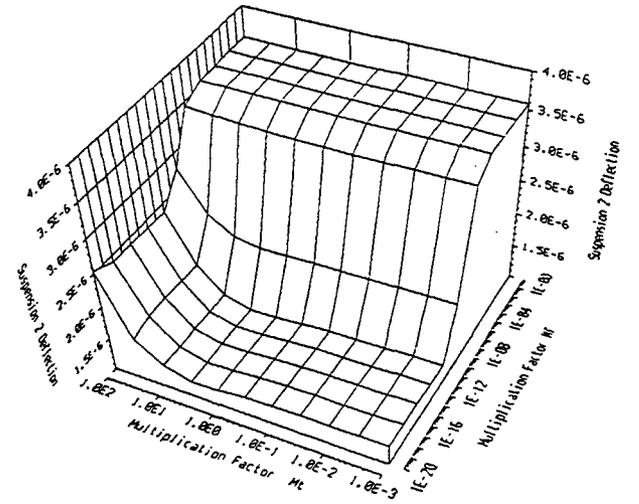
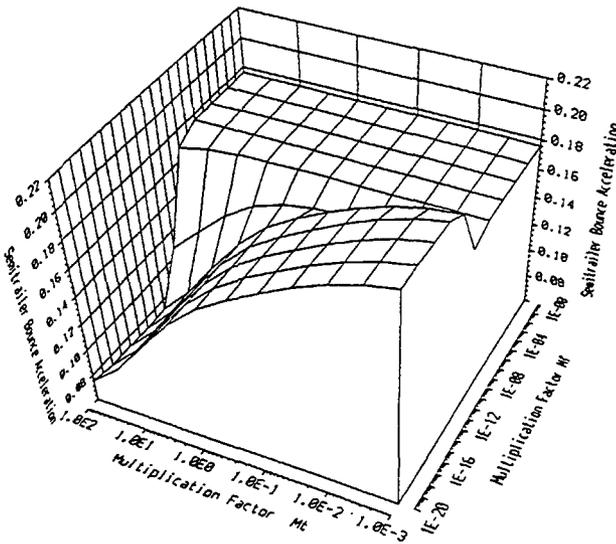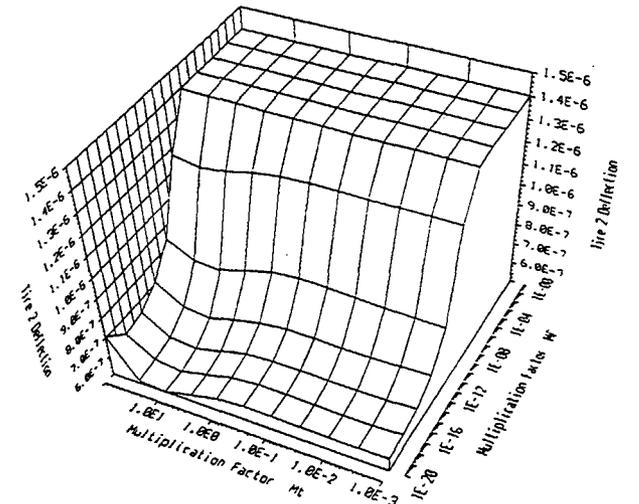Figure 6. Influence of variations in weighting coefficients on the mean square suspension 2 deflection

Figure 5a. Influence of variations in weighting coefficients on the mean square semitrailer bounce acceleration response

Figure 7. Influence of variations in weighting coefficients on the mean square tire 2 deflection

FREQUENCY RESPONSE CHARACTERISTICS OF THE ACTIVELY SUSPENDED VEHICLE

A study of the resonant behavior and the ride quality necessitates the analysis of frequency spectra of the response characteristics of the different active suspension schemes. The frequency response characteristics of the active suspensions schemes (AS1 to AS5) subjected to excitations arising from a rough road are obtained and compared to the frequency response characteristics of the "optimum" passive suspension [9].

Figures 8 to 11 present the PSD of the penalized variables for the five active and the "optimum" passive suspensions. All the five active suspension schemes offer improved ride quality and better cargo safety almost in the entire frequency range, as shown in Figures 8 and 9. Substantial benefits can be observed with all of the 5 active suspension schemes especially at low frequencies, which is a critical frequency region in ride quality evaluations. At higher frequencies, near the axles resonant frequencies around 10 Hz, the performance characteristics of the active suspensions are similar to those of the "optimum" passive suspension. The active suspensions, irrespective of the weighting coefficients, do not provide a significant improvement in the vertical and pitch responses of the tractor and semitrailer sprung masses near the wheel hop frequencies. This behavior of the active suspension is also supported by the studies reported in [16,17], which were based on a 2 DOF vehicle model. It has been shown that a single point, corresponding to the wheel hop frequency, exists on the sprung mass body acceleration response. This point is known as the invariant point and can not be changed regardless of the suspension scheme used [16,17].

The magnitude of relative deflection response across all the five axle suspensions also reduces considerably when proposed active suspensions are employed, as shown in Figure 10. The active suspensions, however, yield high suspension deflection at low frequencies, especially below 0.9 Hz, as illustrated in Figures 10a to 10c. The actuators are constantly trying to correct for any changes in the tractor, semitrailer and wheel motions using feedback signals related to all the states of the vehicle. The actuator forces are thus continuously adjusted to correct for the changes in the motion. Mathematically, the transfer functions for the active suspension deflections do not necessarily reach zero at low frequencies as in the case of a passive suspension. This limitation of active suspensions is also reported in the literature [16,18-21]. Figures 11a to 11e illustrate the frequency response spectra of the tire forces due to active and passive suspensions. Active suspension systems yield significant reduction in dynamic tire forces at low frequencies. The reduction of the dynamic tire forces, however, is not evident near wheel hop frequencies (near 10 Hz). The suspension scheme AS5 yields the most significant reduction in the dynamic tire forces in the entire frequency range. This active suspension scheme is thus desirable when the design objectives involve a specific emphasis on the pavement fatigue caused by dynamic tire loads transmitted to the road.
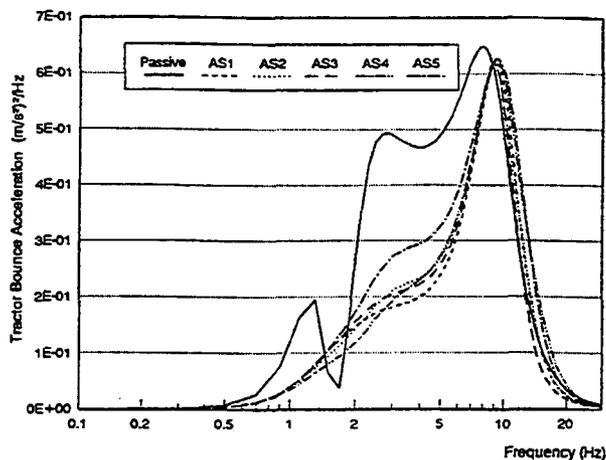

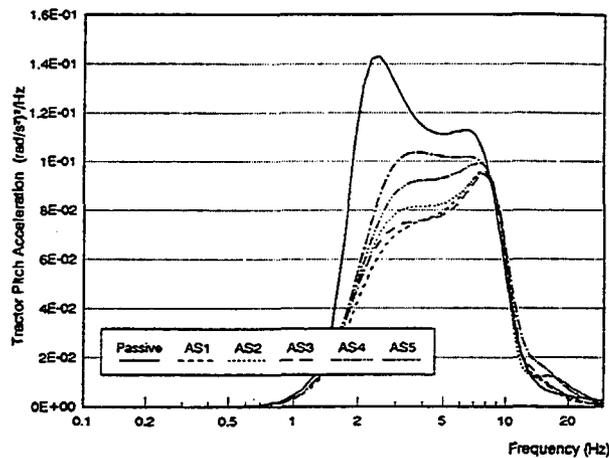Figure 8a. PSD of tractor bounce acceleration response

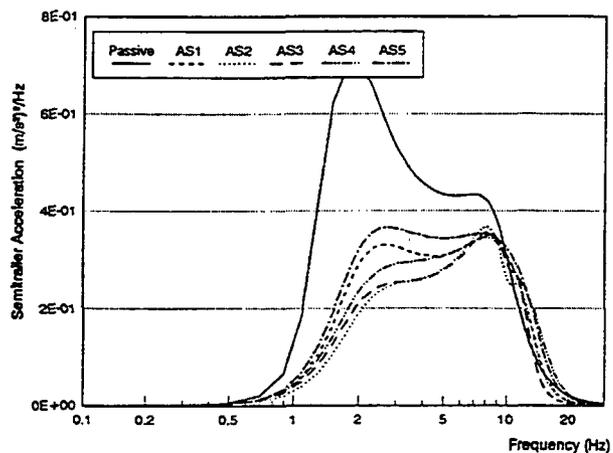Figure 8b. PSD of tractor pitch acceleration response

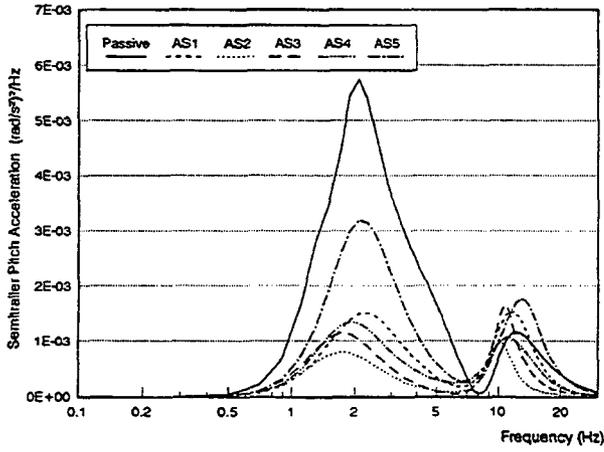Figure 9a. PSD of semitrailer bounce acceleration response

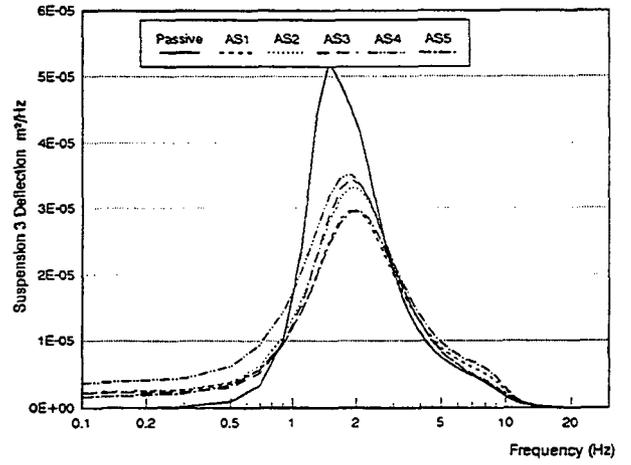Figure 9b. PSD of semitrailer pitch acceleration response

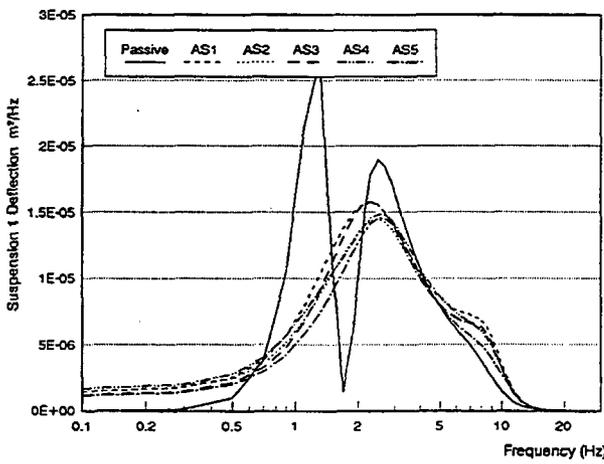Figure 10c. PSD of suspension 3 deflection response

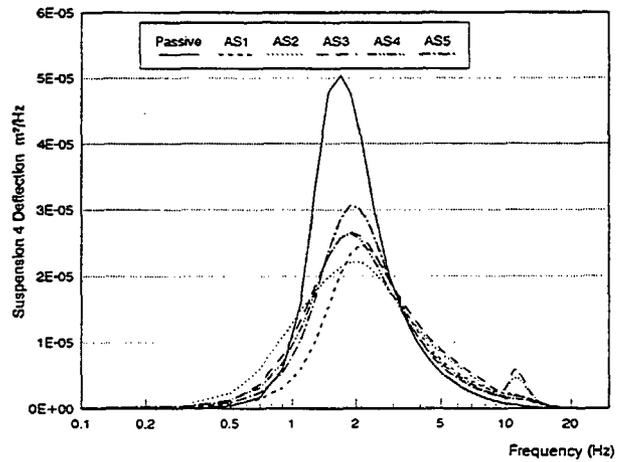Figure 10a. PSD of suspension 1 deflection response
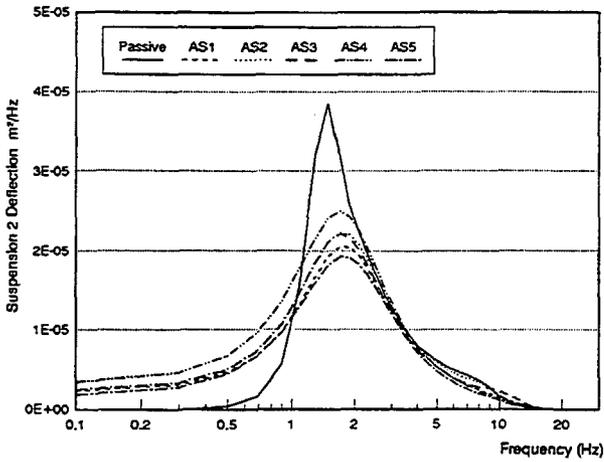
Figure 10d. PSD of suspension 4 deflection response

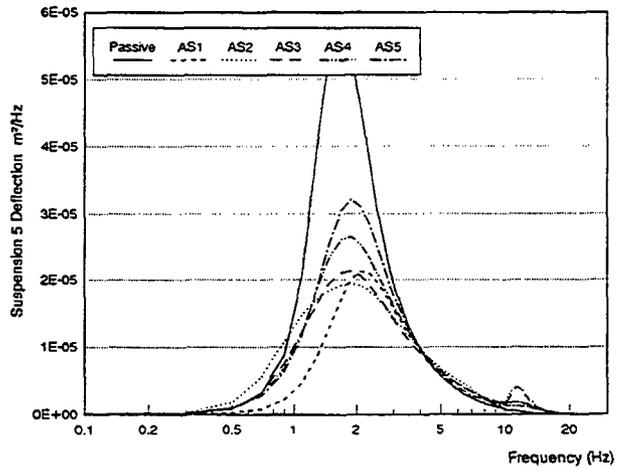Figure 10b. PSD of suspension 2 deflection response

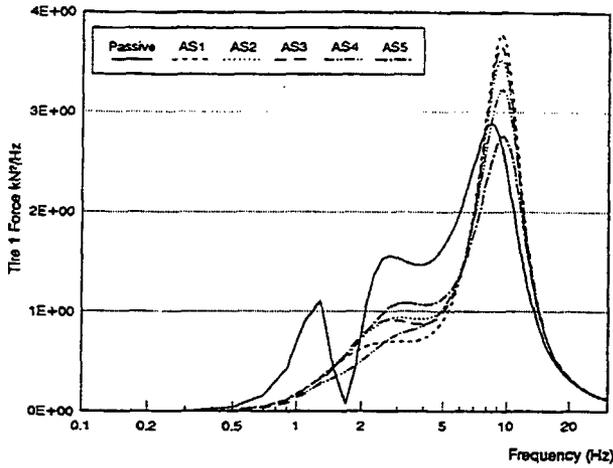Figure 10e. PSD of suspension 5 deflection response
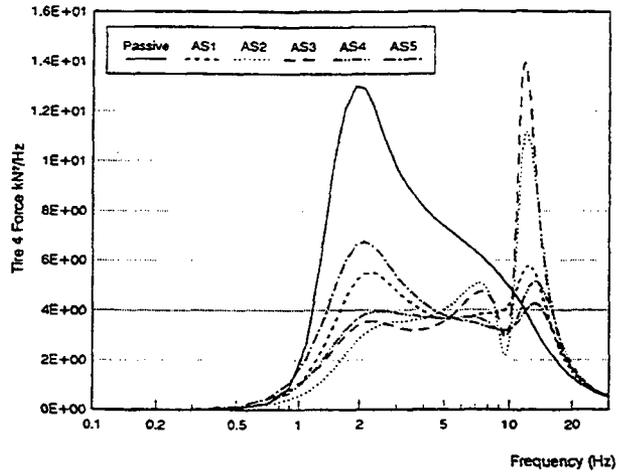
Figure 11a. PSD of tire 1 force response

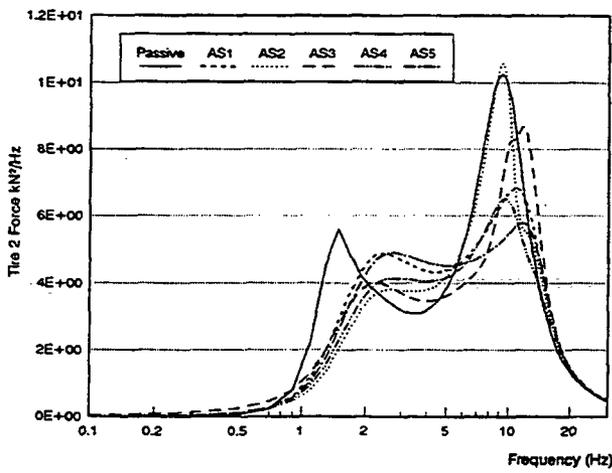Figure 11b. PSD of tire 2 force response
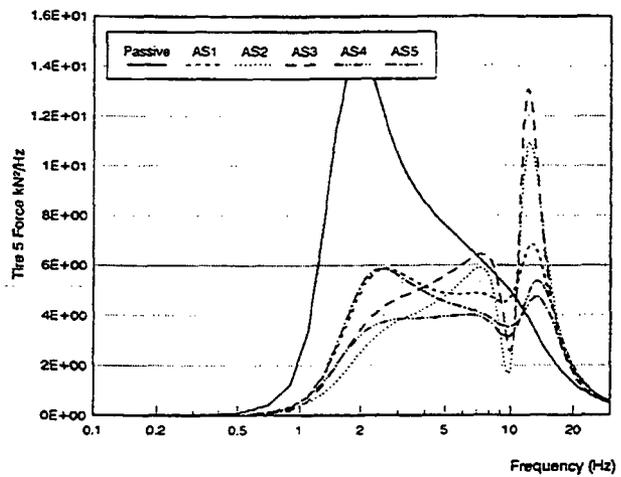

Figure 11c. PSD of tire 3 force response

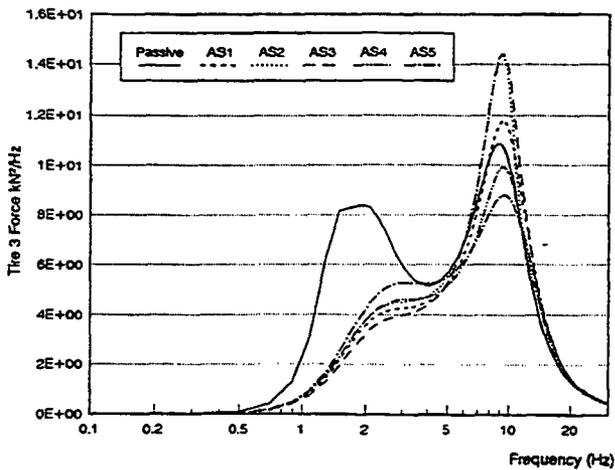Figure 11d. PSD of tire 4 force response

Figure 11e. PSD of tire 5 force response

CONCLUSIONS

A method based on LQG technique is used to design the active suspensions for the tractor-semitrailer vehicle. The second order coupled differential equations of motion of the actively suspended vehicle reduced to first order state-space form are augmented with the road excitation to satisfy the white noise input requirement of the LQG design. The performance objectives of the suspensions comprising the tractor and semitrailer bounce and pitch accelerations and the suspension and tire deflections are expressed in the matrix form as a function of the augmented state vector and the actuator forces. The performance index is then formulated,

and each of its components is determined separately. While the passive elements do not affect the overall performance of the active suspension, the properties of the passive components are chosen to ensure minimum actuator force requirements. Five different sets of weighting coefficients are derived to achieve the design compromise for the active suspension, and the corresponding reduction/increase in the expected values of the penalized variables are investigated. The results clearly illustrate that the active suspensions yield improved performance related to all the design objectives. The suspension performance related to ride quality, cargo safety and tire forces can be further improved by relaxing the rattle space requirements.

ACKNOWLEDGMENT

REFERENCES

1. J. K. Hedrick, *"Some Optimal Control Techniques Applicable to Suspension System Design,"* ASME Publication, 73-ICT-55, 1973.
2. A. G. Thompson, *"Optimal and Suboptimal Linear Active Suspension for Road Vehicles,"* Vehicle System Dynamics, Vol. 13, No. 2, 1984, pp. 61-72.
3. D. R. Guenther and C. T. Leondes, *"Synthesis of a High-Speed Tracked Vehicle Suspension System. Part I: Problem statement, suspension structure and decomposition, and Part II: Definition and solution of the control problem"* IEEE Trans. on Automatic Control, Vol. AC-22, No. 42, 1977, pp. 158-172.
4. A. Hac, *"Suspension Optimization of a 2 DOF Vehicle Model Using Stochastic Optimal Control Technique,"* Journal of sound and vibration, Vol. 100, No. 33, 1985, pp. 343-357.
5. F. Oueslati and S. Sankar, *"Performance of a Fail-Safe Active Suspension with Limited State Feedback for Improved Ride Quality and Reduced Pavement Loading in Heavy Vehicles,"* 1992 Truck & Bus Meeting & Exposition Toledo, Ohio.
6. F. Oueslati and S. Sankar, A. K. W. Ahmed, *"Limited-State Active Suspension for Improved Ride Quality in Rail Vehicles Subjected to Vertical Track Irregularities,"* The International Symposium on Technological Innovation in Guided Transportation, Sep. 28-30, 1993, Lille, France.
7. D. G. Stephens, *"Comparative Vibration Environment of Transportation Vehicles,"* Passenger Vibration in Transportation Vehicles, ASME AMD, Vol. 24, 1979, pp. 59-72.
8. R. D. Ervin and G. Yoram, *"The Influence of Weights and Dimensions on the Stability and Control of Heavy Duty Trucks in Canada,"* University of Michigan Transportation Research Institute, Report No. 86-35, July 1986.

9. F. Oueslati and S. Sankar, *"Optimization of a Tractor-Semitrailer Passive Suspension Using Covariance Analysis Technique"* 1994 International Truck & Bus Meeting & Exposition, Seattle, Washington, Nov. 7-9, 1994, pp. 47-61.
10. A. E. Bryson and Y. C. Ho, *"Applied Optimal Control, Optimization, Estimation and Control,"* New York: John Wiley and Sons, 1975.
11. V. Rajnikant Patel and N. Munro, *"Multivariable System Theory and Design,"* International Series on Systems and Control, Vol. 4. 1st ed. Oxford, Eng. New York: Pergamon Press, 1982.
12. C. J. Dodds, *"The Laboratory Simulation of Vehicle Service Stress,"* Journal of Engineering for Industry, ASME Transactions, Vol. 96, No. 2, May 1974, pp. 391-398
13. C. J. Dodds and J. D. Robson, *"The Description of Road Surface Roughness,"* Journal of Sound and Vibration, Vol. 31, No. 2, 1973, pp. 175-184.
14. J. D. Robson, *"Road Surface Description and Vehicle Response,"* International Journal of Vehicle Design, Vol. 1, No. 1, 1979, pp. 25-35.
15. J. S. Bendat and A. G. Piersol, *"Random Data: Analysis and Measurement Procedures,"* Whiley-Interscience, 1971, Toronto, Ontario, Canada.
16. A. Hac and I. Youn, *"Optimal Semi-Active Suspension with Preview Based on a Quarter Car Model,"* Transactions of the ASME, Journal of Vibration and Acoustics, Vol. 114, 1992 pp. 84-92.
17. J. K. Hedrick and T. Butsuen, *"Invariant Properties of Automotive Suspensions,"* Proc. International Conference on Advanced suspensions, London Oct. 24-25 1988, pp. 35-42.
18. R. M. Chalasani, *"Ride Performance Potential of Active Suspension Systems-Part I: Simplified Analysis Based on a Quarter Car Model,"*. 1986 ASME Winter Annual Meeting, AMD-Vol. 80, pp. 187-204.
19. R. M. Chalasani, *"Ride Performance Potential of Active Suspension Systems-Part II: Comprehensive Analysis Based on a Full Car Model,"* 1986 ASME Winter Annual Meeting, AMD-Vol. 80, pp. 205-234.
20. A. Hac, *"Optimal Linear Preview Control of Active Vehicle Suspension,"* Vehicle System Dynamics, Vol. 21, 1992 pp. 167-195.
21. T. Butsuen and J. K. Hedrick, *"Optimal Semi-Active Suspensions for Automotive Vehicles: The 1/4 Car Model,"* 1989 ASME Winter Annual Meeting, Vol. 13, pp. 305-316.