



**Pages 130-146**

## **SIMPLIFIED PROCEDURE FOR DETERMINING LATERAL STABILITY OF HEAVY VEHICLE COMBINATIONS**

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### **ABSTRACT**

The lateral stability of heavy vehicle combinations is a most important part of active vehicle safety and traffic safety. Rearward amplification is a common way of characterising lateral stability. This measure describes the amplification of the movements of the trailing units relative to the movements of the motor vehicle during some kind of steering manoeuvre. Rearward amplification may be described in terms of yaw velocity gain or lateral acceleration gain. In ISO 14791 [1,2] three different test procedures for determining rearward amplification are described. The first method uses a pseudo random steer input and yields a full representation of the system gain in the frequency domain. The other two methods describe ways to determine rearward amplification for specific, realistic lane-change manoeuvres. The results from these three methods will differ. The results, obtained in the time domain, provide only the composite gain of the system as results from the distributed frequency content of the specific lane change performed in the test, but the results in the frequency domain provide a complete description of the frequency dependency of the rearward amplification

One of the time-domain methods is a path-following lane change. This is particularly difficult to perform with sufficient reliability and reproducibility. Even if the motor vehicle is following the path within the rather narrow tolerance limits as specified in the standard test procedure, the test can still produce substantially different results depending on how the path is chosen within the tolerance band [3]. The other method defines a single sine-wave steer input. This method typically requires the use of a steering machine.

This paper presents an alternative procedure to estimate rearward amplification in the time domain, that is, for a path-following lane change or for the open-loop, single sine-wave steer manoeuvre - or, indeed, for any arbitrary steering manoeuvre. In this new procedure, transfer functions for the appropriate vehicle motions are first obtained from using the pseudo-random test method. These functions, along with the input signals of specified manoeuvres in the time domain, are used to estimate the vehicle responses in the time domain. The maximum rearward amplification for the specified manoeuvre can then be determined using the estimated time responses.

The advantages of the proposed method are that (i) it is based on the pseudo-random input method which is easy and fast to carry out, (ii) the reliability and reproducibility are better than for path following methods and (iii) no sophisticated test equipment is required. Moreover, tests carried out with heavy vehicle combinations show very good agreement between measured and estimated responses in the time domain.

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## **INTRODUCTION**

Stability of heavy vehicles is of great concern for the traffic safety. The most obvious type of instability is a rollover. Because of the, normally, high position of the centre of gravity in relation to the track, a laden truck can easily reach its rollover limit when cornering on flat surfaces if the speed is too high in relation to the path curvature. This is the biggest difference, with respect to stability, between large trucks and passenger cars. Cars normally never roll over on flat surfaces since the adhesion limit of their tyres is reached before this occurs.

Heavy vehicle combinations, however, are much more complicated with respect to stability. In various manoeuvres they may respond with more or less violent oscillations of different kinds. This oscillatory behaviour may be hazardous both to the vehicle itself and to other traffic. The lateral stability of heavy vehicle combinations is therefore of great concern for and is a most important part of active vehicle safety and active traffic safety. Therefore, it is essential to be able to characterise and quantify lateral stability. Several different measures for characterising lateral stability are appropriate for vehicle combinations. Among these are (i) rearward amplification (ii) dynamic offtracking (iii) zero-damping speed and (iiii) yaw damping. This paper is concerned with procedures for determining the rearward amplification.

## **REARWARD AMPLIFICATION**

When steering manoeuvres are performed with vehicle combinations, (i.e., vehicles consisting of several vehicle units connected with articulation joints), oscillations are often excited. The more rearward units tend to oscillate with larger amplitudes than the more forward ones. This behaviour is called rearward amplification and is a commonly used way of characterising lateral stability.

Rearward amplification normally refers to the gain between the motor vehicle and the last trailer and it may be expressed either in terms of yaw velocity gain or lateral acceleration gain. These two gain factors are not identical but are similar. While yaw velocity gain indicates how large the offtracking is, lateral acceleration gain indicates the rollover risk.

In ISO 14791 three different procedures to determine rearward amplification are described [1,2]. The first one uses a pseudo random steer input and the other two specify different ways of performing a lane change resulting in rearward amplifications for specific, more realistic manoeuvres. The results from these methods will differ.

### **Pseudo random steer input**

In performing this test, both the frequency and amplitude of the steering input must be varied randomly. The frequency content must be adequate and the ratio of maximum and minimum steering-wheel angle in the spectrum should not exceed 4:1. This method yields a full representation of the system gain in the frequency domain and thus provides a complete description of the frequency dependency of the rearward amplification. One prerequisite of this test is that it be conducted within the linear operating range of the vehicle. This is verified by determining the coherence function, which may not be less than 0.95 in the relevant frequency range. The procedure is easy and fast to perform as the whole frequency range may be covered in one test run.

### **Single sine-wave steer input**

This method could be described as an open-loop, single lane change. One full period sinusoidal steering input is applied to the steering wheel, followed by a period of neutral steering-wheel position. This creates a lane change of the vehicle. The heading of the vehicle after the manoeuvre is, however, likely to be different from the initial heading. The test is repeated at different frequencies in order to find the maximum rearward amplification. The gain of the yaw velocity or

the lateral acceleration is measured in the time domain. The result is sensitive to the shape of the sine-wave input and therefore a steering machine has to be used in order to get reliable results.

### Single sine-wave lateral acceleration input

In this test procedure, the driver has to follow a prescribed path which gives a lateral acceleration of the first vehicle unit corresponding to one full period of a sinusoid. The path is defined in the global x,y-plane in equation (1) for given maximum lateral acceleration,  $a_y$ , test speed,  $v$ , and frequency,  $f$ .

$$y = \frac{a_y}{(2\pi f)^2} \left[ 2\pi f \frac{x}{v} - \sin \left( 2\pi f \frac{x}{v} \right) \right] ; \text{ for } 0 \leq x \leq \frac{v}{f} \quad (1)$$

Rearward amplification is determined from the time signals of the motor vehicle and the last trailer. The test must be repeated using paths representing different frequencies of lateral acceleration in order to determine the maximum gain.

The results obtained in the time domain using either of the lane-change procedures are different from those obtained in the frequency domain because they provide only the composite gain of the system as results from the distributed frequency content of the specific lane change performed in the test [2]. Consequently the results from the two lane-change tests may also be different from each other due to the fact that the frequency content in the two manoeuvres may be different.

The path-following lane change is a closed loop test and has the largest similarity to real manoeuvres. It is however also the most difficult one to perform. Even when the driver succeeds in following the path within the  $\pm 0.15$  m tolerance interval that is specified in the standard test procedure, the test may produce substantially different results depending on how the path is chosen within the tolerance band [3].

### ESTIMATION OF TIME RESPONSE

In order to determine rearward amplification in some kind of manoeuvre, it is necessary to know the time responses. They are normally measured in the time domain, but especially for path-following test procedures it may be difficult to perform the manoeuvre accurately enough and to obtain reliable results.

If the vehicle exhibits reasonably linear behaviour in the operating range, there is a way to overcome these difficulties. The transfer functions (ie frequency response functions) of the vehicle system contain full information about its dynamic characteristics. By using the transfer functions between the input and the responses of the first and last vehicle units together with the input of the actual manoeuvre, time responses can be estimated and rearward amplification in the time domain for the manoeuvre can be determined. The transfer functions are obtained from a random steer input test.

The responses to be estimated in the standard test procedures are either the yaw velocities or the lateral accelerations of the first and the last vehicle units. These two response signals,  $y_1(t)$  of the first unit and  $y_2(t)$  of the last unit, and the input signal,  $x(t)$ , are measured in a random steer test and the transfer functions  $H_1(i\omega)$  and  $H_2(i\omega)$  are calculated. The complex transfer functions between the responses and the input may be defined according to equation (2) and (3).

$$H_1(i\omega) = Y_1(i\omega) / X(i\omega) \quad (2)$$

$$H_2(i\omega) = Y_2(i\omega) / X(i\omega) \quad (3)$$

where  $X(i\omega)$ ,  $Y_1(i\omega)$  and  $Y_2(i\omega)$  are the fourier transforms of  $x(t)$ ,  $y_1(t)$  and  $y_2(t)$  respectively.

The input may be either the lateral acceleration of the path, in path-following tests, or the steering-wheel angle, in open loop tests. This input in the time domain,  $x(t)$ , is converted to the frequency domain by fourier transformation.

$$X(i\omega) = \text{FFT}(x(t)) \quad (4)$$

By multiplying the input in equation (4) with the transfer functions from equations (2) and (3) the responses are obtained in the frequency domain according to equations (5) and (6)

$$Y_1(i\omega) = H_1(i\omega) * X(i\omega) \quad (5)$$

$$Y_2(i\omega) = H_2(i\omega) * X(i\omega) \quad (6)$$

The responses can now be converted to the time domain with the inverse fourier transform.

$$y_1(t) = \text{IFFT}(Y_1(i\omega)) \quad (7)$$

$$y_2(t) = \text{IFFT}(Y_2(i\omega)) \quad (8)$$

In the case where the gain of the lateral acceleration is to be determined in the path-following lane change, the procedure can be shortened somewhat. The response of the first unit,  $y_1(t)$ , is then equal to the input,  $x(t)$ , because the lateral acceleration is measured at the path-following point. Consequently the transfer function  $H_1(i\omega)$  is unity and need not be calculated.

The rearward amplification is defined as the quotient between the maximum absolute peak values of  $y_2(t)$  and  $y_1(t)$  for the specific manoeuvre. In order to find the maximum rearward amplification, the frequency of the input shall be varied.

The flow chart in [figure 1](#) illustrates the procedure schematically.

## PERFORMED TESTS

In order to validate the previously outlined procedure for estimating the time responses, tests were made with a heavy vehicle combination. Tests were carried out both in the frequency(Hz) and in the time(s) domains. The measured variables were steering-wheel angle(deg), yaw velocity(deg/s) of the first unit, yaw velocity(deg/s) of the last unit, lateral acceleration( $\text{m/s}^2$ ) on the front axle of the first unit and lateral acceleration( $\text{m/s}^2$ ) at the centre of gravity of the last unit.

## Vehicle specification

The vehicle used was a truck and full trailer combination with a gross combination weight of 60 tonnes and a total length of 25.25 metres. It is based on the modules in EU-directive 96/53/EC [4], 7.82 m and 13.6 m, and is therefore a so-called modular combination. It consists of a three axle truck with one driven axle, a two axle dolly and a three axle semitrailer. Principal measures and axle loads appear in [figure 2](#). A real vehicle combination is shown in [figure 3](#).

## Pseudo random steer input

In order to determine the transfer functions between the responses and the inputs, a random steer test was performed. The responses were lateral acceleration and yaw velocity of the first and last vehicle units. The inputs were either lateral acceleration of the first vehicle unit or steering wheel angle. The test speed was 80 km/h and the total duration time over which the transfer functions were averaged was 20 minutes. The coherence functions for all the transfer functions were calculated in order to make sure that the coherence was at least 0.95 in the relevant frequency range. An example of the modulus of a transfer function, measured between lateral acceleration of the first and last vehicle units, is shown in [figure 4](#). It appears that the maximum gain occurs at a frequency around 0.5 Hz. The coherence function, in the same figure, is well above 0.95 in the frequency range of interest.

## Path-following lane change

The lane change was made at four frequencies, 0.3 0.4 0.5 and 0.6 Hz. (The word frequency in this context is inadequate. What is meant is the inverse of the time of the single sine wave pulse. This pulse has rather a wide frequency content.) They were chosen in order to cover a frequency range which comprises the frequency at which the peak in the transfer function occurs. The paths were calculated according to equation (1). The maximum lateral acceleration for each path was set to  $2 \text{ m/s}^2$  and the test speed was 80 km/h. The path was marked with painted marks on the ground. In order to verify that the motor vehicle was following the path a video camera, mounted under the vehicle, was used. After some exercise, one out of five runs was accepted. At least five accepted runs were made for each test case.

## RESULTS

The measurements were evaluated in order to make comparisons between measured and estimated responses. Rearward amplifications in different test procedures were also calculated and compared.

### Validation

In order to validate the procedure for estimation of time responses, the time history of the steering-wheel angle from an arbitrary closed-loop (i.e., not path-following) lane change was used as input signal. (See figure 5.) The lateral acceleration and yaw velocity responses were estimated by using this input signal and the transfer functions in accordance with the description in figure 1. Figures 6 and 7 show measured and estimated time responses for the first and the last vehicle units. Lateral acceleration is shown in figure 6 and yaw velocity in figure 7. As shown in the figures the yaw velocity response of the last vehicle unit is underestimated as is the first peak of both yaw velocity and lateral acceleration. One possible cause for this discrepancy may be that the transfer functions and the validation measurements were made on different occasions with different weather conditions. As a whole, however, the agreement between measured and estimated signals is very good.

### Measured and estimated responses

The rearward amplification in a path-following lane change, where the acceleration of the path is one full period of a sinusoid, is calculated from the time histories of the estimated responses in the first and last vehicle units. Examples of estimated lateral acceleration and yaw velocity responses for the input frequency 0.5 Hz are shown in figure 8. The rearward amplification factors were obtained by dividing the maximum absolute peak values by each other. They are plotted in figure 9 versus the input frequency.

The accuracy of the estimated values depends on the confidence of the transfer function estimates. The closer to unity the coherence is and the larger the number of averages is, the smaller the error of the transfer function estimate is. The coherence was generally not below 0.97 and the number of averages was 60. This gives a normalized random error which is less than 0.02 [5].

The rearward amplifications were calculated in the same way for the measured responses. Mean values and standard deviations for each test case are shown in table 1.

Frequency (Hz)	LATERAL ACCELERATION		YAW VELOCITY	
	Mean rearward amplification	Standard deviation	Mean rearward amplification	Standard deviation
0.3	1.25	0.07	1.44	0.09
0.4	1.38	0.06	1.55	0.04
0.5	1.34	0.05	1.61	0.07
0.6	1.38	0.03	1.61	0.02

**Table 1. Measured rearward amplifications**

The mean values of the measured rearward amplifications are plotted together with the estimated rearward amplifications versus input frequency in the diagram in [figure 9](#).

It appears from the comparison in figure 9 that the measured rearward amplification factors fall within the same range as the estimated ones and that the maximum rearward amplifications are approximately the same. While the estimated values have a pronounced peak, as can be expected, the measured amplification factors have a less systematic pattern. For lateral acceleration the measured values seem to follow the estimated curve well for the three lowest frequencies, while the highest frequency yields the highest amplification. The measured yaw velocity values have an increasing trend and also here the maximum amplification is obtained at the highest frequency. The reason for that is believed to be that the tolerance interval of the path allows quite a large deviation from the ideal path [3], especially for the higher input frequencies. This may dramatically change the frequency content of the input, and the excitation content is shifted towards the resonance frequency of the vehicle system, thus increasing the amplification. It can be seen in table 1 that the standard deviation for some test cases is very low. This does however only show that the systematic errors, inherent in the test method, can be well reproduced.

## Different test procedures

In ISO 14791 there are three possible ways of determining the rearward amplification. They all normally give, and shall be expected to give, different results. If different vehicle combinations are compared, the ranking of the vehicles will however be the same for each method.

The rearward amplifications obtained from the test in the frequency domain and in the time domain are compared. The rearward amplification from the random test is the modulus of the transfer function between the responses of the last and first vehicle units. The rearward amplifications from the path-following lane change and the single sine-wave steer input are calculated from the estimated time responses. The estimations are made for input frequencies between 0.3 and 0.7 Hz with an interval of 0.05 Hz. The lateral acceleration and yaw velocity rearward amplifications with the different test procedures are shown as a function of frequency in [figure 10](#).

Because this vehicle combination is well damped, the peak in the transfer function is wide and the gain is rather low. The rearward amplification in the two lane change manoeuvres is however significantly lower, which it should be expected to be. With a less well-damped vehicle, the difference would have been larger. Another difference is that the maximum rearward amplification occurs at frequencies which are lower than the resonance frequency seen in the transfer function. This depends on the distribution of the frequency content in the responses. For this vehicle the path-following lane change yields a larger maximum rearward amplification, especially for the lateral acceleration, than the sinusoidal steer input. These relationships may however vary with the vehicle characteristics.

## CONCLUSIONS

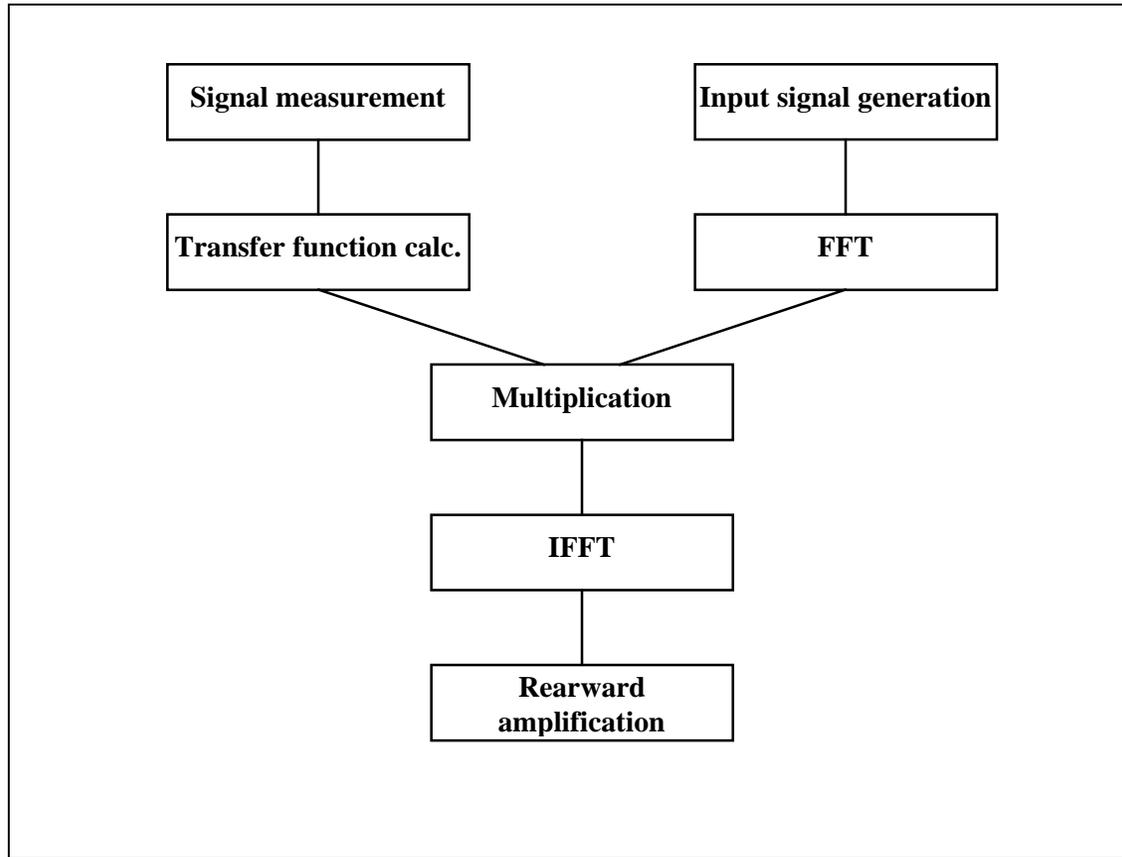
Rearward amplification is an important characteristic of lateral stability of vehicle combinations. The pseudo random steer input test gives a full representation of the system gain in the frequency domain and thus provides complete information about the frequency dependency of the rearward amplification. It may however be desirable to relate the rearward amplification to a specific, realistic type of manoeuvre, for example a path-following lane change. The traditional way of performing such tests is to carry out the manoeuvre in real life and measure the responses in the time domain. This is, however, a cumbersome and laborious procedure, and it is difficult to achieve satisfactory accuracy and reproducibility. It is therefore suggested that an alternative procedure, based on results from the pseudo random steer input test, be used to estimate the time responses for the desired manoeuvre. The advantages of this approach are that it is easy and fast to conduct and that no sophisticated test equipment in order to perform the manoeuvre is required. Moreover, this procedure shows very good validity.



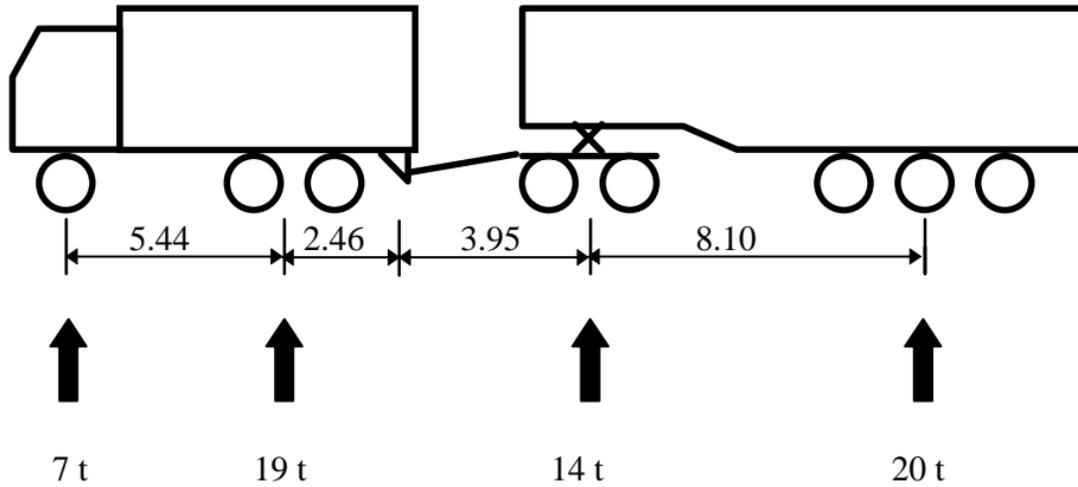
## REFERENCES

1. ISO 14791 - Heavy commercial vehicle combinations and articulated buses - Lateral stability test procedures.
2. John Aurell and Chris Winkler, "Standard test procedures for the lateral stability of heavy vehicle combinations", Road Transport Technology-4, Proceedings of the fourth international symposium on heavy vehicle weights and dimensions, Ann Arbor ,USA, 1995.
3. J. Preston-Thomas and M. El-Gindy, "Path compliance in lane-change tests designed to evaluate rearward amplification", Road Transport Technology-4, Proceedings of the fourth international symposium on heavy vehicle weights and dimensions, Ann Arbor ,USA, 1995.
4. EU COUNCIL DIRECTIVE 96/53/EC .
5. Julius S. Bendat and Allan G. Piersol: Engineering Applications of Correlation and Spectral Analysis. 1980 by John Wiley & Sons.





*Figure 1. Procedure for estimation of time responses*



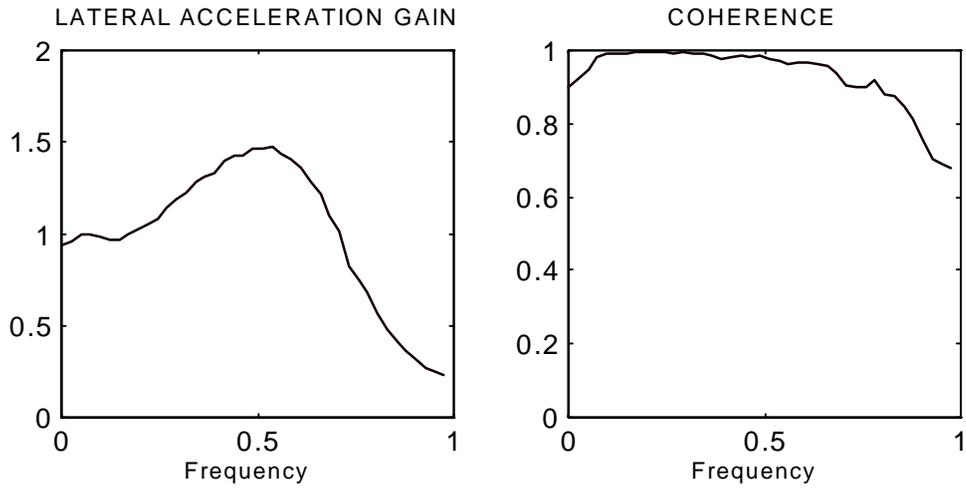
*Figure 2. Combination of truck, dolly and semitrailer, 25.25 m long.*





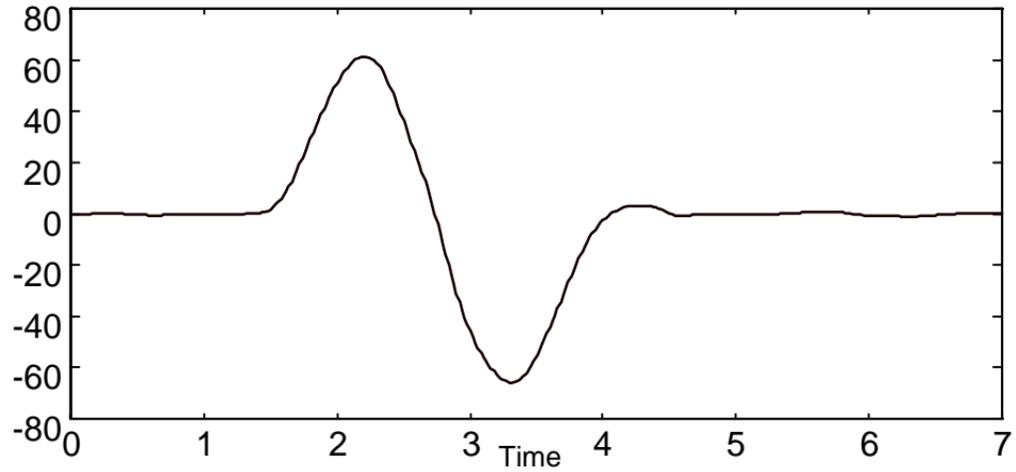
*Figure 3. Modular combination*





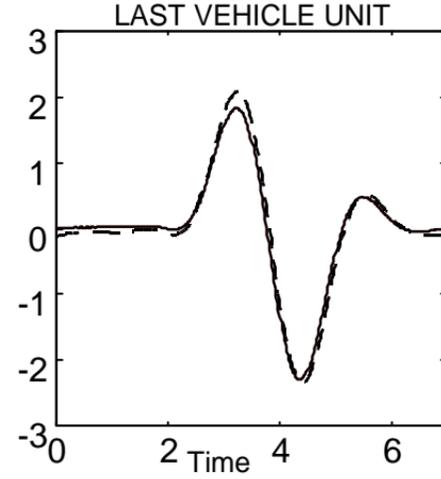
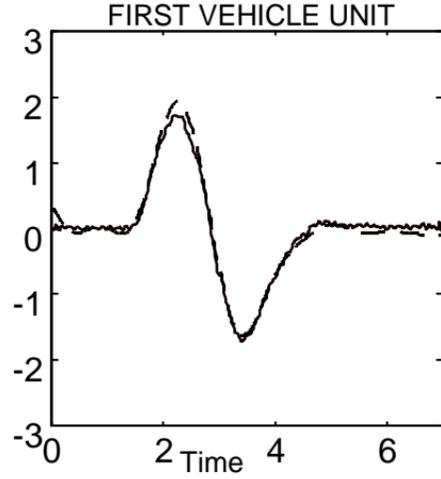
*Figure 4. Transfer function and coherence function between first and last vehicle units*





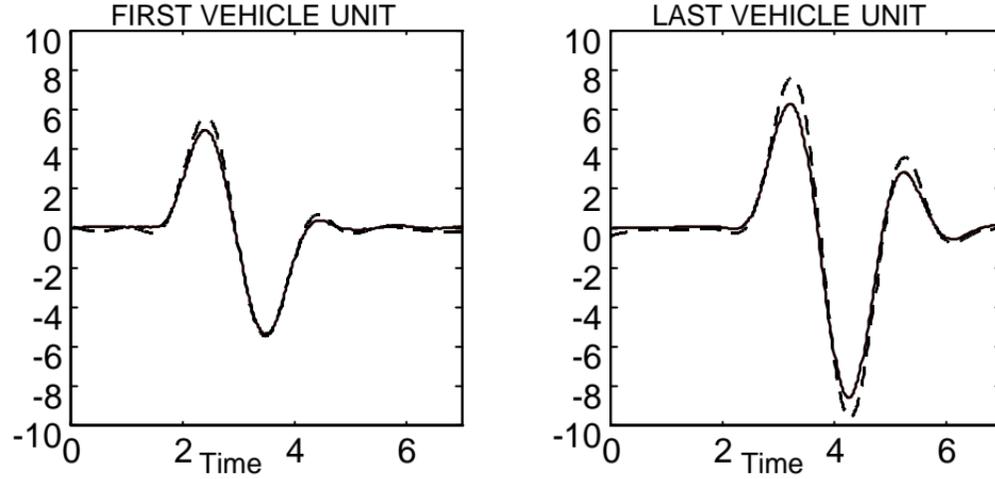
*Figure 5. Steering-wheel angle from a lane change*





*Figure 6. Measured( \_ \_ \_ \_ \_ ) and estimated( \_ \_ \_ \_ \_ ) lateral acceleration responses*





*Figure 7. Measured( \_ \_ \_ \_ ) and estimated( \_ \_ \_ \_ ) yaw velocity responses*



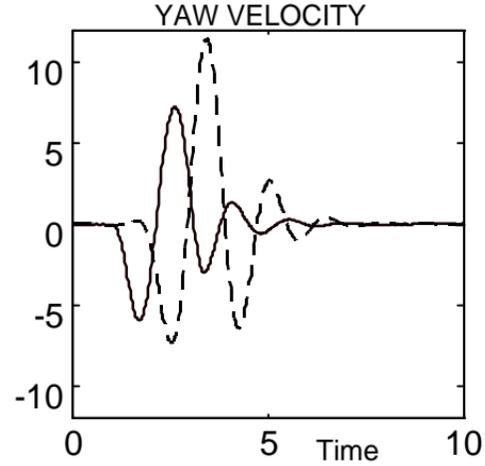
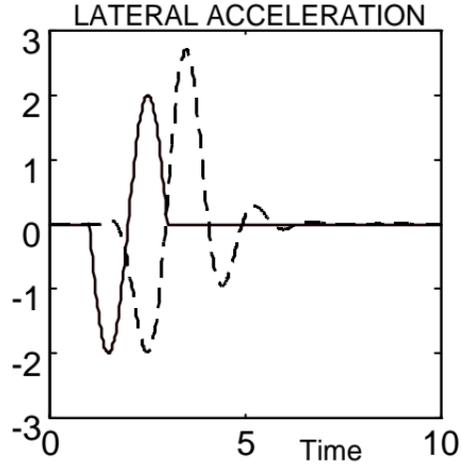
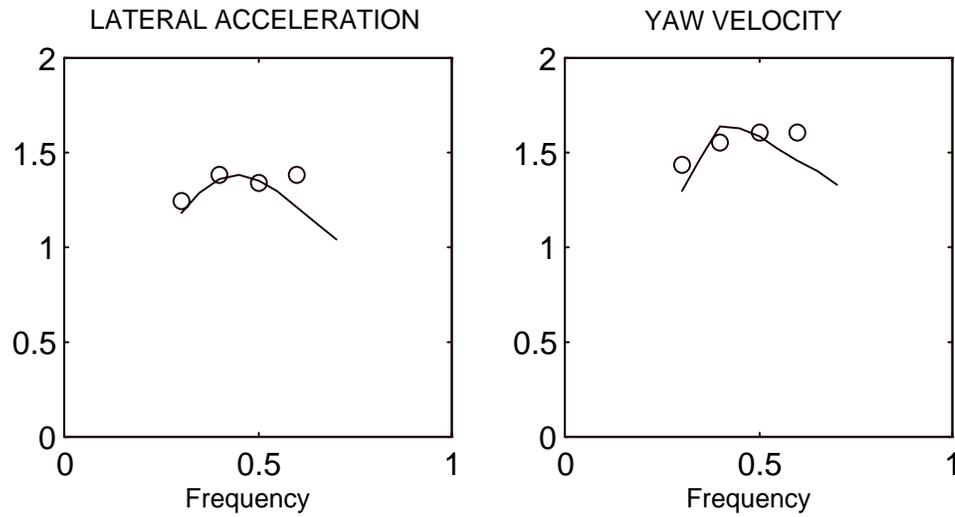


Figure 8. Estimated responses of first ( \_\_\_\_\_ ) and last ( \_ \_ \_ \_ ) vehicle units in a path-following lane change

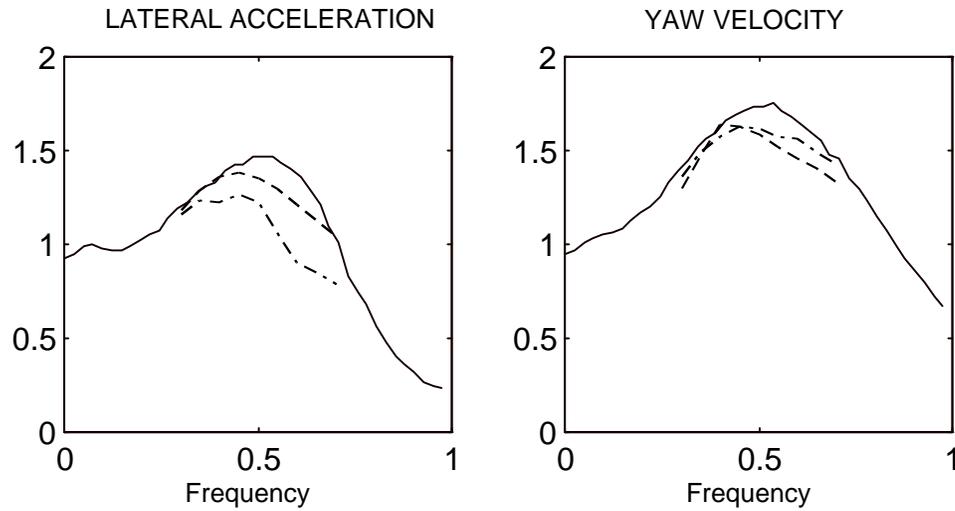
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[Content](#)



*Figure 9. Measured (  $\circ$  ) and estimated ( \_\_\_\_\_ ) rearward amplification in a path-following lane change*





*Figure 10. Rearward amplifications in random steer input ( \_\_\_\_\_ ), path-following lane change ( - - - - ) and single sine-wave steer input ( \_ . . . . )*

[Return to paper](#)

[Content](#)