TRAILER STEERING: AN AUSTRALIAN RESEARCH PERSPECTIVE AND APPLICATION FOR BY-WIRE CONTROL

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ABSTRACT

Dangerous goods vehicles, in particular those carrying fluid loads, have been identified as needing to meet stringent performance requirements within the Performance Based Standards (PBS) developed by the National Roads and Transport Commission in Australia. The opportunity to control the lateral dynamics of these multiply-articulated vehicles with active steering systems deployed on the trailers has been identified by a number of groups worldwide, but research into the dynamics, and modern control strategies to modify these dynamics, is sparse. RABiT at the University of Melbourne identified a need to address the gaps in this knowledge and has proposed a prototype vehicle with trailer steer control that could gain unrestricted route access under PBS with significant mass increases when compared to current, similarly configured vehicles.

This paper illustrates the modelling approach taken for the development of a trailer steer control algorithm, in which the hitch angle between successive articulated units is allowed to be large (in contrast to most previous work where small angle assumptions are made). It is shown by comparison with simulations performed with a complex, nonlinear 3D model that a single-track, yaw-plane model with linear tyre behaviour (but large hitch angles) is able to capture the dynamics effects important for trailer tracking control (including the onset of jackknife).

A partial feedback-linearising controller is developed, which seeks to make the axle of each semitrailer unit track the path of the preceding hitch point. It is shown that considerable improvements in low-speed offtracking can be made, and that rearward amplification of lateral accelerations at high-speeds can be reduced.

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1 INTRODUCTION

The Australian transport industry, specifically the heavy trucks sector, is planning on entering a new phase of regulation by the second decade of this century. A new set of vehicle standards is to be tested here for the first time anywhere in the world, and we will either be the first to benefit, or the first to suffer.

Under these newly envisaged Performance Based Standards it will be possible to certify a vehicle for provisional operation on any of Australia's roads without ever having built it. These vehicles will be initially 'proofed' through simulation alone, able to gain an initial nod of approval from the regulator before being prototyped. There is one caveat to this approval, in that final certification depends on demonstrating agreement between what was predicted, and what the vehicle is finally capable of delivering when measured against these standards.

In anticipation of the introduction of these standards the researchers at RABiT within the University of Melbourne decided to pursue a line of investigation into just what sort of productivity gains would be possible under the new regulations; where it is understood that Australia's road freight is to double within the next 15 years, so an emphasis on increased payload per vehicle would be favourably viewed. A novel vehicle design was proposed as part of the preliminary work, and a PhD program was suggested to investigate the potential realisation of a type of truck that would employ actively steerable axles on its following trailers. A payload capacity increase of at least 15% would be achievable with this type of vehicle.

Subsequent reviews of the literature identified this as an area where little work had been undertaken, but where some significant new thrusts were being made, in particular within the U.K. at Cambridge University (Jujnovich and Cebon 2002) and at the University of California, Berkeley (Tai, Wang et al. 2001; Tai and Tomizuka 2003). Similar theoretical work had also been undertaken on buses at Eindhoven University of Technology (de Bruin and Bosch 1999; de Bruin, Damen et al. 2000; de Bruin 2001). However, none of those approaches seemed to grasp the problem from the uniquely Australian perspective, where our new standards will make high-axle-load, multiply-articulated heavy vehicles desirable. As a consequence, a significant advantage in performance and payload capability is achievable by introducing tractor path tracking.

The principle of this type of tracking is depicted in Figure 1, where ideally the axle centre of the subsequent semitrailer (shown at two instants by yellow dots) would attempt to follow the in-plane path of the tractor's steer axle centre (red dot) along the dashed line. Our proposed system of control, which we will discuss later, assumes that a large part of this tracking can be achieved by linking together a series of independent trailer steer axles that only try to track their own trailer's preceding hitch point. The solid line is an illustration only of what the comparable hitch point tracking system might do.

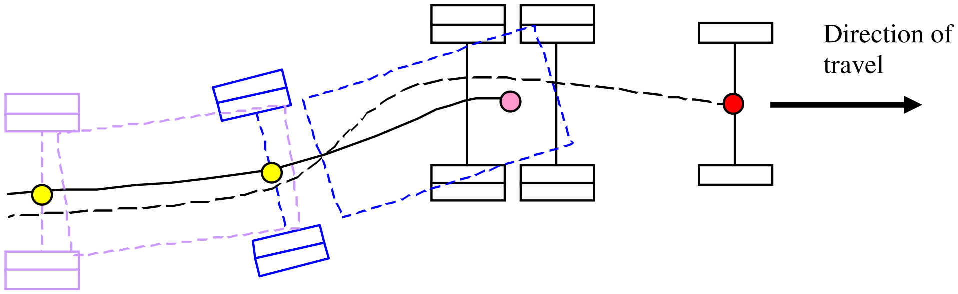

Figure 1. Comparison of combination trailer axles (yellow dots) tracking the preceding hitch point (pink dot), rather than the tractor's steer axle (red dot).

2 DYNAMIC MODEL

We first investigated the possible advantages of various levels of modelling detail for the purpose of capturing the relevant lateral dynamics of the tractor-trailer combination. In most previous work the vehicle's roll, pitch and bounce behaviour is assumed to be insignificant with regard to the lateral response characteristics, resulting in the seemingly ubiquitous planar single-track or 'bicycle' model of a vehicle (Figure 2). Recent work by Louca, Rideout et al. (2004), examining in detail the contributions toward the faithful reproduction of the lateral dynamic behaviour of models, has confirmed that for lateral accelerations up to 0.2g yaw plane models are adequate.

This type of model is quite popular and successful in passenger vehicle control studies, and seems to have naturally migrated to heavy vehicle studies despite the significant differences in roll response and suspension characteristics between the two platforms (Fancher 1989). There are two notable exceptions in the literature where roll and perhaps pitch degrees of freedom have been included: Jujnovich and Cebon (2002) and Gafvert and Lindgarde (2004). In both cases, the extra degrees of freedom were included for braking studies.

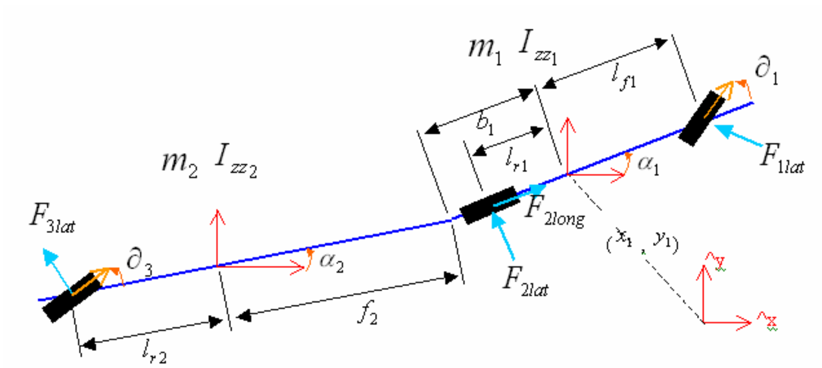

Figure 2. Yaw plane 'bicycle' model of tractor-semitrailer .

It is common in studies of the stability and control of the lateral dynamics of articulated vehicles to linearise the equations of motion about a straight-running trajectory (e.g., Fancher 1989; de Bruin and Bosch 1999; Pacejka 2002; Dunn, Heydinger et al. 2004). Besides the usual assumption of small steer and tyre-slip angles, this requires that the articulation angle of each trailer with respect to its pulling unit is also small. These assumptions mean that cornering at both low and high speeds must be for small road curvatures. At high speeds this is certainly the case for normal safe driving practice, but at low speeds the restriction of small hitch angles can become limiting. For our models we have decided to retain and evaluate the nonlinearity associated with large hitch angles.

Two types of single-track model have been developed for the present study:

1. A Dymola multi-body simulation model, for performing numerical ‘experiments’ and displaying the results in graphs and animations, and
2. A set of nonlinear state-space equations (derived below), to use in formulating the tracking controller.

Both models incorporate the same assumptions, and should be dynamically equivalent:

- We assume that the pitch and roll of the vehicles are insignificant with regard to their influence on the lateral dynamics we wish to control, so essentially we do not look at braking of the system.
- The forward speed is assumed constant (precisely so in the analytical model; maintained approximately so in the simulation model with closed-loop control of drive forces).
- The tyre forces have been treated as an average on each axle, implying symmetry in loading and response, and small effects from camber, caster and pneumatic trail have been ignored, including steering caster effects.
- Self-aligning moments on the tyres have been neglected.
- Tyre lateral forces are assumed to be proportional to the tyre slip angle. Thus, tyre slip angles are assumed to be small (less than 20 deg, say).

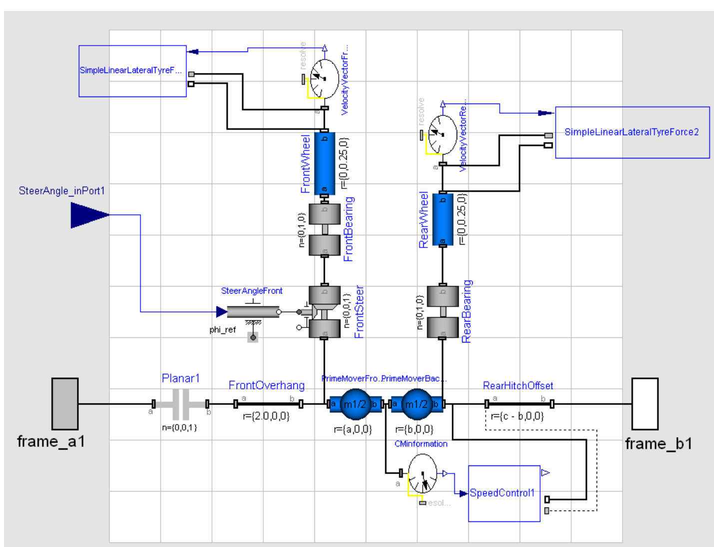

Figure 3. Dymola yaw plane model of tractor-semitrailer

2.1 Dymola model

Dymola (Dynasim 2004) is a multi-physics, multi-engineering suite of encapsulated models based on the open-source Modelica modelling language. The interactions between components are allowed to be acausal and hybrid (both continuous and discrete). We have used Dymola in two ways: (a) to implement and solve the differential equations of the analytical yaw plane model, and (b) to 'construct' a multi-body yaw plane tractor-semitrailer model by interconnecting pre-packaged elements from the Dymola library: masses, joints and tyre force function generators. Figure 3 illustrates a portion of the latter model.

2.2 Analytical model

In this modelling approach we follow closely the recent derivations using Lagrange's Equations by DeBruin (2001) and in the PATH project (Chen and Tomizuka 1995; Chen and Tomizuka 1997).

Given a vehicle comprising any number n of articulated units, it can be shown that if we let

$$\mathbf{x}_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \cdots$$

be the locations in world coordinates of the centres of gravity (CG) of the tractor, the first trailer, and so on for subsequent trailers, then the location of any (the j-th, say) tractor or trailer can be written, using geometrical relationships in the yaw plane, by the following equation:

$$\begin{aligned}\mathbf{x}_j &= \mathbf{x}_1 - \sum_{i=1}^{j-1} \begin{pmatrix} \cos\alpha_i \\ \sin\alpha_i \end{pmatrix} \cdot b_i - \sum_{i=2}^{j} \begin{pmatrix} \cos\alpha_i \\ \sin\alpha_i \end{pmatrix} \cdot b_i \\ &= \mathbf{x}_1 - \sum_{i=1}^{j} \begin{pmatrix} \cos\alpha_i \\ \sin\alpha_i \end{pmatrix} \cdot (b_i + f_i), \quad \text{where } b_j = f_1 = 0,\end{aligned}$$

and consequently

$$\dot{\mathbf{x}}_j = \dot{\mathbf{x}}_1 - \sum_{i=1}^{j} \begin{pmatrix} -\sin\alpha_i \\ \cos\alpha_i \end{pmatrix} \cdot (b_i + f_i) \cdot \dot{\alpha}_i.$$

See Figure 2 for definitions of the geometric parameters.

The kinetic energy T of the assembly of units may then be summarised as

$$T = \tfrac{1}{2} \sum_{j=1}^{n} \begin{pmatrix} \dot{\mathbf{x}}_j^T & \dot{\alpha}_j \end{pmatrix} \mathbf{M}_j \begin{pmatrix} \dot{\mathbf{x}}_j^T & \dot{\alpha}_j \end{pmatrix}^T, \text{ where } \mathbf{M}_j = \begin{pmatrix} m_j & 0 & 0 \\ 0 & m_j & 0 \\ 0 & 0 & I_j \end{pmatrix}.$$

Choosing generalised coordinates $\mathbf{q} = \begin{pmatrix} x_1 & y_1 & \alpha_1 & \alpha_2 & \cdots & \alpha_n \end{pmatrix}^T$ we can apply Lagrange's equations of motion,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial q_k} = \tau_k, \quad k = 1, 2, \ldots, n+2, \tag{1}$$

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where $\boldsymbol{\tau} = \begin{pmatrix} \tau_1 & \tau_2 & \cdots & \tau_{n+2} \end{pmatrix}^T$ are the generalised forces doing work on the system (related to the tyre forces for our vehicle system). This requires that we derive a Jacobian matrix $\mathbf{N}_j$ relating the velocity in world coordinates to the generalised coordinates for each vehicle unit:

$$\begin{pmatrix} \dot{\mathbf{x}}_j^T & \dot{\alpha}_j \end{pmatrix}^T = \mathbf{N}_j \dot{\mathbf{q}}.$$

Then, we can write the kinetic energy as

$$T = \tfrac{1}{2} \dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q}) \dot{\mathbf{q}}, \text{ where } \mathbf{M}(\mathbf{q}) = \sum_{j=1}^{n} \mathbf{N}_j^T \mathbf{M}_j \mathbf{N}_j.$$

Lagrange's equations (1) then yield equations of motion of the form

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} = \boldsymbol{\tau}\left(\mathbf{q}, \boldsymbol{\delta}, \mathbf{F}_{\text{tyres}}\right), \tag{2}$$

where $\boldsymbol{\delta}$ is a vector of steer angles, and $\mathbf{F}_{\text{tyres}}$ is a vector of lateral tyre forces and the tractor longitudinal drive force.

For the case of the tractor-semitrailer combination shown in Figure 2, for which $n = 2$, we get:

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$$\mathbf{q} = \begin{pmatrix} x_1 \\ y_1 \\ \alpha_1 \\ \alpha_2 \end{pmatrix}, \quad \mathbf{M}(\mathbf{q}) = \begin{pmatrix} m_1 + m_2 & 0 & m_2 b_1 \sin\alpha_1 & m_2 f_2 \sin\alpha_2 \\ 0 & m_1 + m_2 & -m_2 b_1 \cos\alpha_1 & -m_2 f_2 \cos\alpha_2 \\ m_2 b_1 \sin\alpha_1 & -m_2 b_1 \cos\alpha_1 & m_2 b_1^2 + I_1 & m_2 b_1 f_2 \cos(\alpha_2 - \alpha_1) \\ m_2 f_2 \sin\alpha_2 & -m_2 f_2 \cos\alpha_2 & m_2 b_1 f_2 \cos(\alpha_2 - \alpha_1) & m_2 f_2^2 + I_2 \end{pmatrix}$$

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \begin{pmatrix} 0 & 0 & m_2 b_1 \cos\alpha_1 \cdot \dot{\alpha}_1 & m_2 f_2 \cos\alpha_2 \cdot \dot{\alpha}_2 \\ 0 & 0 & m_2 b_1 \sin\alpha_1 \cdot \dot{\alpha}_1 & m_2 f_2 \sin\alpha_2 \cdot \dot{\alpha}_2 \\ 0 & 0 & 0 & m_2 b_1 f_2 \sin(\alpha_2 - \alpha_1) \cdot \dot{\alpha}_2 \\ 0 & 0 & m_2 b_1 f_2 \sin(\alpha_2 - \alpha_1) \cdot \dot{\alpha}_1 & 0 \end{pmatrix}$$

$$\boldsymbol{\tau}(\mathbf{q}, \delta_1, \delta_3, \mathbf{F}_{\text{tyres}}) = \begin{pmatrix} -F_{1lat} \sin(\alpha_1 + \delta_1) + F_{2long} \cos\alpha_1 - F_{2lat} \sin\alpha_1 - F_{3lat} \sin\left(\alpha_2 + \delta_3\right) \\ F_{1lat} \cos(\alpha_1 + \delta_1) + F_{2long} \sin\alpha_1 + F_{2lat} \cos\alpha_1 + F_{3lat} \cos\left(\alpha_2 + \delta_3\right) \\ l_{f1} F_{1lat} \cos\delta_1 - l_{r1} F_{2lat} - b_1 F_{3lat} \cos(\alpha_2 + \delta_3 - \alpha_1) \\ -(f_2 + l_{r2}) F_{3lat} \cos(\delta_3) \end{pmatrix}$$

For the purposes of control design, we apply a series of transformations to tractor-fixed coordinates, and a reduction in degrees of freedom by assuming that, for the duration of a manoeuvre, the vehicle maintains near constant speed. From (2) we then get a final set of state-space equations of motion which is affine in the trailer steer input δ_3:

$$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}) + \mathbf{g}(\mathbf{z}) \cdot \delta_3 \tag{3}$$

where

$$\mathbf{z}=\begin{pmatrix} y_1' \\ \alpha_1 \\ \theta \\ v \\ r \\ \dot{\theta} \end{pmatrix}, \quad \mathbf{f}(\mathbf{z})=\begin{pmatrix} v \\ r \\ \dot{\theta} \\ f_4(\mathbf{z}) \\ f_5(\mathbf{z}) \\ f_6(\mathbf{z}) \end{pmatrix}, \quad \mathbf{g}(\mathbf{z})=\begin{pmatrix} 0 \\ 0 \\ 0 \\ g_4(\mathbf{z}) \\ g_5(\mathbf{z}) \\ g_6(\mathbf{z}) \end{pmatrix}$$

Here, v and r are the lateral and yaw velocities respectively of the tractor, and y_1' and α_1 are their respective integrals with respect to time, while θ is the hitch angle between tractor and trailer. The functions **f** and **g** are nonlinear functions of the augmented state vector **z**. Putting the equations of motion in the form (3) allows us to consider a feedback linearisation control design approach.

3 MODEL VALIDATION

Recall that the nonlinear equations of motion (3) were derived to provide a basis for development of a trailer steer controller. They do not include load transfer effects, nor tyre nonlinearities. When linearised about a straight-line trajectory they reduce exactly to the linear equations developed by Dunn, Heydinger et al. (2004). As a further validation step, the response predictions of both the nonlinear and linearised versions of (3) were compared with those of a comprehensive, 77-DOF, 3D ADAMS model of a tractor-semitrailer version of the novel vehicle mentioned in the Introduction. The ADAMS model accounts for basic nonlinear tyre behaviour using a Magic Formula handling model for the XZA 315 80 R22.5 tyres with combined slips and including tyre relaxation effects. This tyre model was provided by TNO. The ADAMS model alsoincorporates air spring and auxiliary roll stiffnesses estimated from UMTRI data to capture load transfer effects.

Figure 4 shows the results of a 2-deg step-steer test at 60 km/h. In such a manoeuvre the slip and hitch angles are quite small and, as expected, the linear and nonlinear equations yield almost identical results. Their response predictions also compare favourably with those of the complex ADAMS model, particularly for the hitch angle response. To investigate the effects of large hitch angles, the same comparisons were performed for a number of low-speed, high-curvature turns. Figure 5 shows that, for a 40 deg step-steer turn at 10 km/h, the nonlinear single-track equations (3) successfully predict the jackknife failure exhibited by the ADAMS model. Thus, we believe that the present nonlinear models provide a reasonably accurate representation of the lateral behaviour of a tractor-semitrailer over a wide range of constant speed manoeuvres.

4 CONTROL

As illustrated previously in Figure 1, our control objective is to have the trailer axle track the path of the hitch point, by manipulating the trailer steer angle δ_3. The control approach we have implemented is based on partial feedback linearisation of equation (3). If we construct the control as $\delta_3 = g_6^{-1}\left(-f_6 + v\right)$, the last of equations (3) reduces to $\ddot{\theta} = v$. A pole placement tracking design, with integral action (effectively a PID controller), yields the control law:

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$$\delta_3 = -\frac{f_6}{g_6} + \frac{1}{g_6}\left[\begin{pmatrix} K_p & K_d & K_i \end{pmatrix}\begin{pmatrix} \theta - \theta_r \\ \dot{\theta} - \dot{\theta}_r \\ \sigma \end{pmatrix} + \ddot{\theta}_r\right] \tag{4}$$

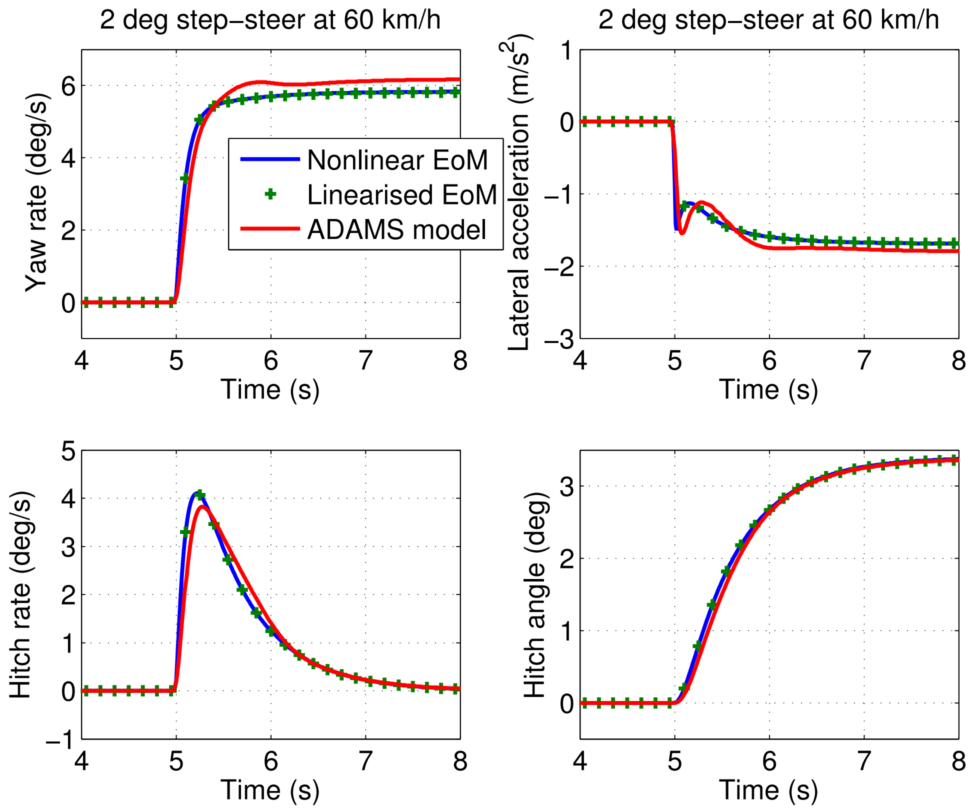

Figure 4. Validation of analytical equations of motion (EoM) against 3D ADAMS model for small hitch-angle manoeuvre.

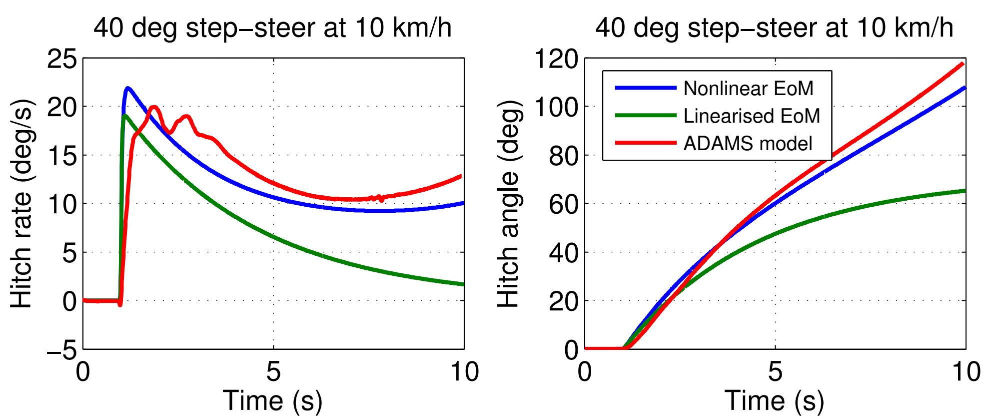

Figure 5. Validation of analytical equations of motion (EoM) against 3D ADAMS model for large hitch-angle manoeuvre.

In equation (4) the reference hitch angle is θ_r, and the integral of the hitch angle error is σ. We have developed an algorithm to reliably compute the reference hitch angle online; using measurements or estimates of the yaw rate of the leading unit, and its lateral velocity at the hitch angle. The performance of the controller in seeking to make the trailer of the tractor-semitrailer described by Dunn, Heydinger et al. (2004) track its own hitch point during a low speed turn is shown by the outputs of the Dymola model in Figure 6. In this figure the tractor drive tyre is represented in red, and the other tyres in green. The centres of mass of the vehicle units are shown as blue spheres, the tyre forces are represented by green vectors, and the path of the hitch point is shown by the blue curve. Without trailer steer control, large tracking errors occur. With the controller (4) activated, the trailer tracks the hitch point reference trajectory (shown in blue) very accurately, despite the very large articulation angle developed. The bandwidth of the steering actuator used in this manoeuvre is approximately 3 Hz, which is higher than strictly necessary for this vehicle. However, if subsequent dolly-trailer units are coupled to the truck, limit cycles can appear during steady state travel if the actuator bandwidth is reduced.

A similar evaluation of controller performance is shown in Figure 7 for one such coupled truck-trailer and dolly-trailer system; the A-coupled vehicle studied by Fancher (1982). For this vehicle we utilise two controllers: one for the tractor, first-trailer articulation, and a second for the following dolly, second-trailer articulation. This separation of control is made possible by the weak lateral-force coupling between tractor-trailer and dolly-trailer units through the A-hitch.

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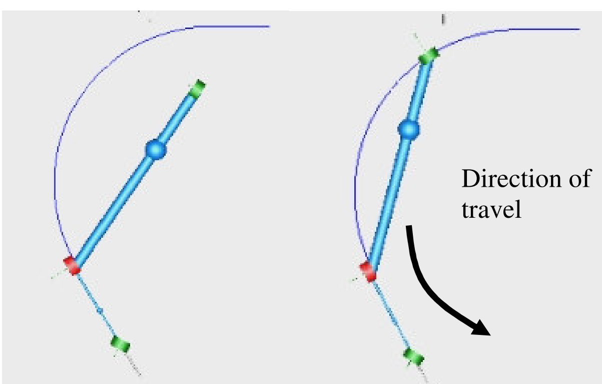

Figure 6. Response of the truck used by Dunn et al. (2004): (left) without trailer steer, and (right) with. Driver input is a 30 deg step-steer at 10km/h.

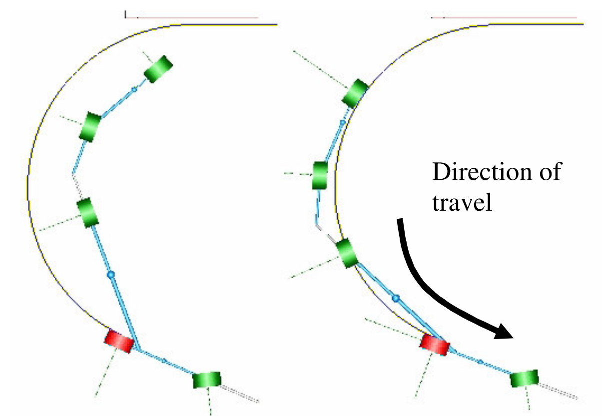

Figure 7. Response of the truck used by Fancher (1982): (left) without trailer steer, and (right) with. Driver input is a 30 deg step-steer at 10km/h.

At high speeds, when hitch angles are small, the controller is not very different from a traditional linear design, and simulation tests show it to be very effective. Figure 8 shows the controlled trailer steer angles for a novel vehicle (similar in layout to that shown in Figure 7), during a lane change manoeuvre at 88 km/h. Only minor steer angles are required at such speeds during lane changes; there is consequently a requirement for the steer actuators to have good precision.

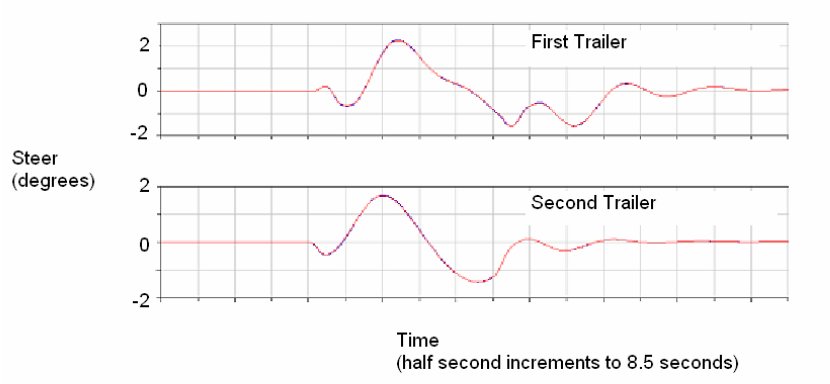

Figure 8. High-speed lane change steer activity in the first and second trailers steer axles on a new vehicle.

Preliminary results indicate that introducing active trailer steer control can be effective in reducing the high rearward amplification of lateral accelerations experienced at high speeds by some poor design vehicles. However, errors and noise in measurements or estimates of the states utilised by the controller can degrade performance markedly. Table 1 displays the rearward amplification levels for the Fancher (1982) A-double in its standard configuration, and then with various design modifications and with 0.05 m/s RMS noise corrupting the estimate of lateral velocity. It is clear that active trailer steer control has significant potential to improve the dynamic behaviour of multiply-articulated vehicles, but considerable attention will be required to address issues of sensor noise and actuator failure in developing practically useful designs.

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Table 1. Effects of trailer steer, design modifications, and sensor noise on rearward amplification of lateral acceleration on the Fancher (1982) vehicle.

Fancher vehicle modifications	Rearward Amplification (ratio of accelerations)
Standard Fancher (1982) double	2.2
+ control on 1st and 2nd trailers	1.8
+ with noisy measure on lateral velocity	3.1
+ no noise, but redesign without overhang on hitch	1.5
+ no noise, without overhang, and with additional steer on dolly axle	1.3

5 FURTHER WORK

Future work on the controller will investigate ways of increasing the robustness of system performance in the face of sensor noise, determining the bounds on stability for the control, and establishing precise requirements for the actuator bandwidth. Additional attention will also be given to the effects of braking and of reversing such a vehicle.

6 CONCLUSIONS

The single-track yaw-plane model of a multiply-articulated vehicle described in this paper has been shown to adequately represent its steady-state and transient behaviour for the purposes of developing a trailer steer control algorithm. Linear tyre behaviour is assumed, but the model makes no explicit small angle assumptions, and in particular allows large articulation angles. For small angles the model reduces precisely to previously published linearised equations of motion.

For large angles, model outputs correlate well with those of a complex 3D simulation model. For a low speed, high curvature turn, the model successfully predicted the onset of a jackknife.

The partial feedback linearising controller based on the present vehicle model is successful in steering multiple trailers to track their preceding hitch points. This leads to considerably improved offtracking behaviour at low speeds with large articulation angles, and significant reductions in rearward amplification at high speed for some vehicle types. Sensor noise has been shown to be an issue, and further work is being performed to improve the controller robustness.

Work is progressing on developing and testing the proposed system on a comprehensive 3D model of a new vehicle design for Australia that could increase payloads by 15% or more.

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