

## ROLLOVER RISK PREVENTION OF HEAVY VEHICLES BY RELIABILITY-BASED ANALYSIS



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### Abstract

The aim of this paper is to develop a reliability-based approach to prevent rollover risk of heavy vehicles. The reliability index is computed to characterize the safe functioning of the vehicle when the driver negotiates a curve in the road. The evaluation is based on a rollover indicator, namely the load transfer ratio between left and right sides of the vehicle. Sensitivity analysis is presented to find the most influential parameters for risk, in order to model them by random variables. After that, the risk is evaluated by computing a reliability index and estimating the probability that the rollover indicator exceeds a threshold. Finally, simulation results are presented to validate the reliability-based approach.

**Keywords:** Truck, Heavy vehicle Dynamics, Rollover, Load transfer Ratio, Reliability theory, Safety index, Probabilistic approach.

### Résumé

Le but de l'article est de prévenir le risque de renversement de poids lourd par une approche fiabiliste. La détermination de l'indice de fiabilité permettra de caractériser le bon comportement du poids lourd lorsqu'il effectue une mise en virage. La prévention est basée sur le rapport de transfert de charge entre les côtés droite et gauche du véhicule. Une analyse de sensibilité est utilisée pour extraire les paramètres les plus influents sur le risque, ces paramètres devenant aléatoires. Le risque est par la suite évalué par un indice de fiabilité et par la probabilité que le l'indicateur du risque dépasse un seuil donné. Enfin, des simulations sont effectuées pour valider l'approche retenue.

**Mots-clés :** Camion, poids lourds, dynamique des poids lourds, renversement, rapport de transfert de charge, théorie de la fiabilité, indice de sécurité, approche probabiliste.

## 1. Introduction

Statistics show that accidents related to heavy goods vehicle (HGV) are more dangerous than those of passenger vehicles Evans et al. (2005), ONISR (2004). While they constitute only 3% of vehicles in traffic, heavy vehicles are involved in 10% of accidents with fatalities. Furthermore, the fatality rate is twice as high when a HGV is implied. Rollover is one of the most frequent accidents (20%) and causes significant damages to the vehicles and injuries to its driver and passengers. Several anti-rollover systems and rollover warning systems were developed to assist or warn the driver. See for example the works of Gaspar et al. (2005), Ackermann et al. (1999), and Dakhllallah et al. (2006).

Most of the current prevention systems have some limitations, because they are based on real time measurements without any prediction of the vehicle dynamics. Moreover, they only use deterministic data while statistical models are needed to give an account of the uncertainties. This leads to a loss of information at the evaluation level. In this case, it is possible to predict free accident situation, while in presence of unanticipated events or uncertainties, for example on the high centre of gravity or on the road elevation, accident may occur. On the other hand, when the HGV behavior and infrastructure are well known, it is possible to be closer to the safety limit while maintaining an acceptable risk level. But with less information, the rollover risk increases, and the driver must reduce its risk by reducing the vehicle speed. Therefore, it is important to take into account the most relevant uncertainties in the rollover risk evaluation.

This paper presents a new method to evaluate rollover risk for heavy goods vehicles, using the systems' reliability theory and tools, in order to provide warning based on statistical information of the driver-vehicle-infrastructure system. As an example, the rollover risk is assessed before a road bend.

Because the HGV dynamics and interactions with the infrastructure induce rather high frequency motions, the time explicitly intervenes in stochastic differential equations. Random processes are needed to model heavy vehicle states. The risk prediction leads to a threshold crossing problem of such processes. To simplify this problem, the dynamic interactions between heavy vehicle and infrastructure are analysed by a deterministic approach, and the uncertainties are modelled by random variables, independent of time. Furthermore, only one value characterizes the rollover risk on the entire prediction interval. In this way, the vehicle state prediction is obtained only by solving ordinary differential equations. This enables us to employ static reliability tools which are widely used in several fields, such as structural reliability. In our study, rollover risk evaluation is based on the maximum of a rollover risk indicator, namely the load transfer ratio (LTR), which corresponds to the load transfer between the left and the right sides of the vehicle.

Besides, it is much more complex to compute the probability if the number of random variables increases. Thus, sensitivity analysis of the driver-vehicle-infrastructure is then dealt with in order to deduce the most influential parameters on the rollover risk. These parameters are then modelled by suitable probability distributions. After that, a reliability based method is developed to compute the reliability index and the corresponding probability of rollover risk, which are obtained with enough accuracy after some iterations. Finally, results are compared and validated by Monte Carlo simulations.

## 2. Heavy Goods Vehicle Model and Rollover Risk Indicator

### 2.1 Heavy Goods Vehicle Modelling

The vehicle studied in this paper is a non-articulated heavy vehicle with two axles. The vehicle model was developed by Ackerman and Odenthal (1999). Therefore, some hypotheses were considered: the vehicle is moving on a flat and level road with a constant longitudinal speed, the roll angle is assumed to be small, and suspension and tire dynamics are assumed to be linear. The corresponding model has three degrees-of-freedom and used to represent the roll and lateral dynamics as shown in Figure 1. The vehicle consists of two bodies. Body 1 which is composed of the two axles is the unsprung mass with mass  $m_1$  and center of gravity CG1. Body 2 is the sprung mass with mass  $m_2$  and centre of gravity CG2. CG1 is assumed to be in the road plane above CG2. Motion equations can be written in the linear form:

$$M\ddot{q} + D\dot{q} + Kq = Su \quad (1)$$

where

$$M = \begin{bmatrix} m & 0 & -hm_2 \\ 0 & Jz & 0 \\ -hm_2 & 0 & Jx + h^2m_2 \end{bmatrix}, K = \begin{bmatrix} 0 \\ 0 \\ c - m_2gh \end{bmatrix}, S = \begin{bmatrix} \mu c_f \\ \mu c_f l_f \\ 0 \end{bmatrix},$$

$$D = \begin{bmatrix} \frac{\mu(c_f + c_r)}{v} & \frac{\mu(c_f l_f - c_r l_r) + mv^2}{v} & 0 \\ \frac{\mu(c_f l_f - c_r l_r)}{v} & \frac{\mu(c_f l_f^2 + c_r l_r^2)}{v} & 0 \\ 0 & -hm_2 v & d \end{bmatrix}$$

$q = \left[ \int v_y dt, \psi, \phi \right]$  is the configuration vector containing lateral translation, yaw motion and roll motion of the vehicle.  $u$  is the steering angle and  $v$  the longitudinal speed. The values of the considered parameters are given in table 1.

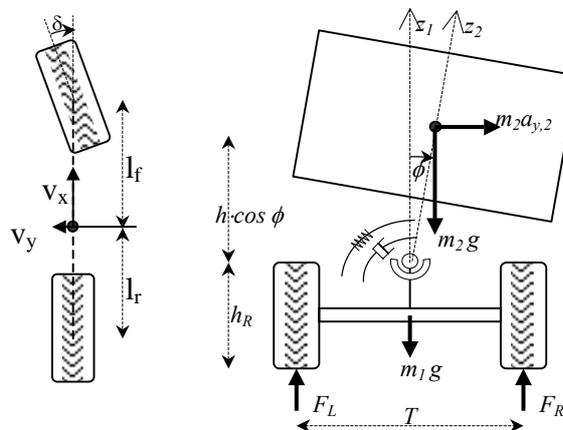


Figure 1 – Heavy vehicle model with 3 DOF.

**Table 1** – Mean values and 95% confidence intervals of the heavy vehicle parameters.

	Description	Mean	Confidence interval (95%)
$l_f$	Distance front axle to CG (m)	2.0	0.1
$l_r$	Distance rear axle to CG (m)	1.5	0.1
$\bar{T}$	Average track width (m)	1.86	0.02
$c_f$	Front cornering stiffness (kN/rad)	582	58
$c_r$	Rear cornering stiffness (kN/rad)	783	78
$\mu$	Road adhesion coefficient	1	0.2
$h_R$	Height roll axis over ground (m)	0.68	$\approx 0$
$h$	Distance CG2 to roll axis (m)	1.15	0.2
$c$	Roll stiffness of suspension (kNm/rad)	457	45.7
$d$	Roll damping of suspension (kNms/rad)	100	10
$m$	Vehicle mass (kg)	14300	2860
$m_2$	Sprung mass (kg)	12480	2500
$J_x$	Roll moment of inertia (kgm <sup>2</sup> )	25.000	5000
$J_z$	Yaw moment of inertia (kgm <sup>2</sup> )	35.000	7000
$g$	Gravity acceleration (m/s <sup>2</sup> )	9.81	$\approx 0$

## 2.2 Rollover Risk Indicator

Rollover risk evaluation is based on a load transfer metric, namely load transfer ratio (LTR), that estimates the difference in tire normal forces acting on each side of the vehicle. LTR can be defined by establishing the balance of vertical forces and roll moments on CG1. The resulting expression of this indicator is given by:

$$LTR = \frac{F_R - F_L}{F_R + F_L} = \frac{2m_2}{m \cdot \bar{T}} \left( (h_R + h \cos \phi) \frac{a_y}{g} + h \sin \phi \right) \quad (2)$$

$F_L$  and  $F_R$  are normal forces acting on respectively the left and the right sides of the vehicle.

When LTR is equal to 0, the HGV has stable roll dynamics. The risk becomes high as this indicator goes towards  $\pm 1$ . Both extreme values characterize wheel lift-off. So, this indicator gives a necessary but not sufficient condition for rollover accident. The probability of rollover risk is then defined by:

$$P_{risk} = P(|LTR_{max}| > R_{lim}) \quad (3)$$

$R_{lim}$  is chosen in (0,1] according to a desired safety level and  $LTR_{max}$  denotes the maximum of LTR during the prediction period.

### 3. Probabilistic Modelling of HGV Parameters

This section presents a sensitivity analysis of the parameters. The most influential ones are taken as random variables in the reliability-based evaluation of rollover risk.

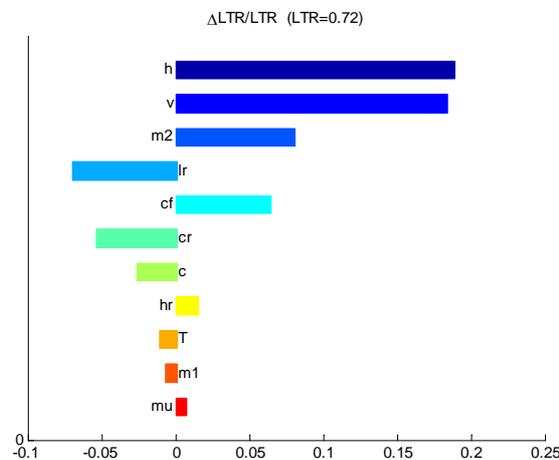
The effect of parametric uncertainties on rollover risk is analysed. For each simulation, only one parameter varies during the cornering and rollover risk is evaluated by the maximum of LTR. The sensitivity of  $LTR_{\max}$  with respect to the vector of independent parameters  $p$  is given by:

$$S = \frac{\partial LTR}{\partial p}, \quad (4)$$

these quantities being numerically evaluated. In order to use equation (4) which involves independency between parameters, the following relations are taken into account:

- $l = l_f + l_r$  ; where  $l$  is invariant.
- $m = m_1 + m_2$  ; where  $m$  is invariant.
- The moment inertia is assumed to depend linearly on the corresponding mass:  $J = m \cdot R^2$  where  $R$ , the radius of gyration, is invariant.
- Parameters  $c_f$ ,  $c_r$  and  $\mu$  are assumed to be independent.

Figure 2 shows some values of the variation coefficient of the maximum of LTR ( $\Delta LTR_{\max} / LTR_{\max}$ ) with respect to each model parameter. The parameters' variations correspond to the values of the confidence intervals given in table 1. The vehicle speed is 15m/s and the steering angle is about  $3^\circ$ .



**Figure 2** – Variation coefficient of  $LTR_{\max}$  for a positive variation of each parameter.

From an analysis based on several driving situations (each being characterized by a speed and a steering angle profile), the influential parameters to be considered as random variables are: the height of the centre of gravity  $h$ , the longitudinal speed  $v$ , the sprung mass  $m_2$ , the longitudinal position of CG ( $l_r$  or  $l_f$ ), and the cornering stiffness ( $c_f$  and  $c_r$ ). These parameters are either known a priori or estimated in real time. However, uncertainties and estimation errors are modelled by random variables with suitable distributions. In this study only  $h$  and  $v$  are assumed to be random with normal distributions. We intend to validate the reliability-based approach to evaluate the vehicle rollover risk.

## 4. Rollover Prevention by Reliability Index

### 4.1 Reliability Method Principle

In order to quantify rollover risk, it is necessary to define a safety margin, namely limit state function, which delimit the safety domain of the heavy vehicle. The limit state function is defined as:

$$g(x) = R_{\text{lim}} - |LTR_{\text{max}}(x)| \quad (5)$$

We express the probability of the risk  $P_{\text{risk}}$  as:

$$P_{\text{risk}} = P(g(X) < 0) \quad (6)$$

where  $X$  is the vector of the  $p$  random parameters and  $g$  is the mapping from  $R^p$  into  $R$  defined in equation (5).

Let the set  $D_f = \{x \in R^p / g(x) < 0\}$  be the unsafe domain. The probability of equation (6) can be obtained by integrating, over the unsafe domain, the joint probability density  $f_X(x)$  of the random vector  $X$ , or by integrating the probability density  $f_Y(y)$  of the random mapping  $Y = g(X)$  as follows:

$$P_f = \iint_{D_f} f_X(x) dx = \int_{-\infty}^0 f_Y(y) ds \quad (7)$$

The first integral, being multidimensional, is numerically complex to be estimated accurately. The second one is a single integral, but it requires the law of the random variable  $g(X)$ , which is often unknown. In order to avoid integration, several methods have been developed: simulation methods and approximation methods. The first ones are based on Monte Carlo method whose computational cost is prohibitive. The second ones are based on approximations of the limit state function  $g(x)$ . These last methods reduce greatly the computational cost. The reliability method used in this study consists in:

1. transforming the physical random vector  $X$  into a centered and normed Gaussian random vector  $U$ :  $U = T(X)$ . The most frequently used transformations  $T$  are Rosenblatt, Nataf, Paloheimo, Rackwitz-Fissler transformation “Melchers (2005)”. The limit state surface can be expressed in the new space as  $H(u) = 0$ , where  $u$  is a realization of  $U$ . The rollover risk probability will be given by:

$$P_{\text{risk}} = P(H(U) < 0); \quad (8)$$

2. approximating the limit state surface by a tangent hyper-plane (FORM method) at the unsafe point  $P^*$  that have the highest probability. High order methods are also developed to overcome nonlinearities of the limit state function “Zhao et al. (2001)”.

In this framework (points 1. and 2.), the probability (8) is equal to  $\Phi(-\beta)$ , where  $\Phi(\cdot)$  is the cumulative distribution function of the centered and normed Gaussian law, and  $\beta$  is the reliability index.

Several reliability indexes were proposed in the literature starting by Rjanitzyne index in 1950 and Cornell in 1970. Hasofer and Lind (1974) present a complete definition of the index: it corresponds to the distance between the origin of the normalized space and the point  $P^*$ . It is obtained by solving the minimisation problem:

$$\beta = \min(\|u\|^2) \text{ under the constraints } H(u) = 0 \quad (9)$$

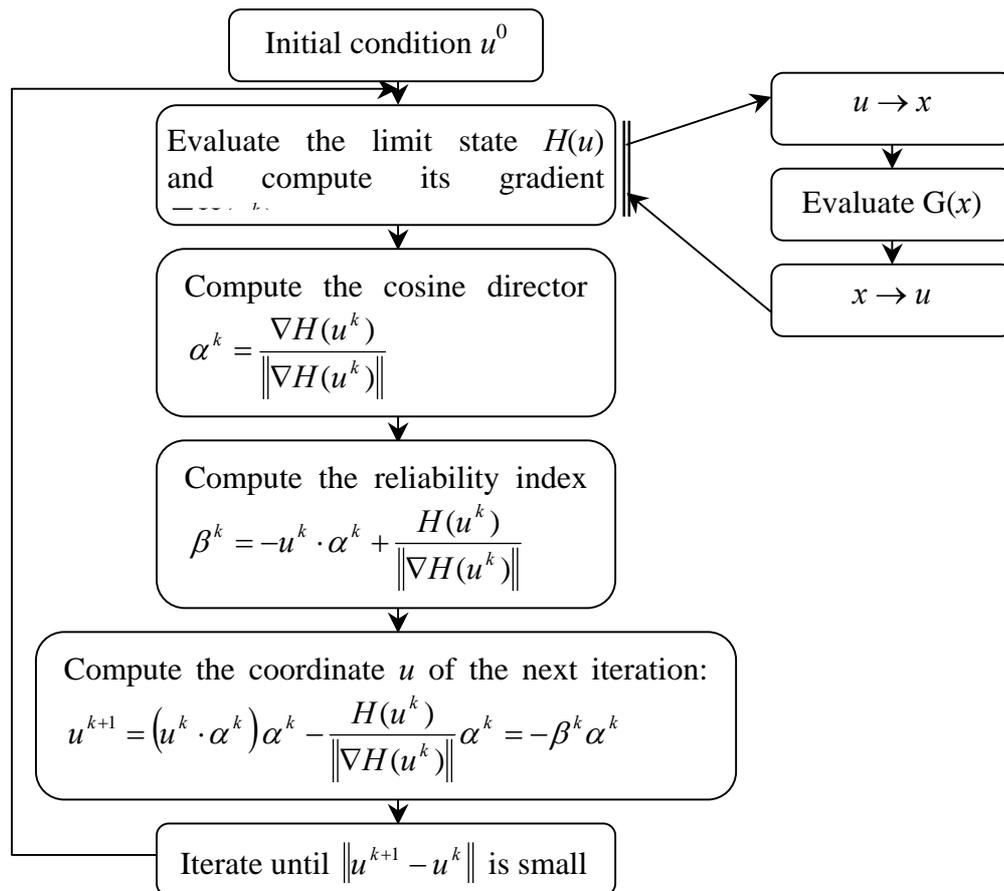
$P^*$  is the point of the normalized space that achieves the above minimum.

#### 4.2 Hasofer-Lind-Rackwitz-Fiessler Algorithm

The Hasofer-Lind-Rackwitz-Fiessler (HRLF) algorithm “Lemaire (2005)” is a first order optimization algorithm to estimate  $\beta$ . From the point  $P^k$  at the  $k^{\text{th}}$  iteration in the normalized space, and after developing Taylor series of the limit state function  $H(u)$  at the point  $P^k$ , we obtain a tangent hyperplane of the limit state. The point  $P^{k+1}$  that belongs to the limit state satisfies the following constraint:

$$H(u^{k+1}) = H(u^k) + \nabla H(u^k) \cdot (u^{k+1} - u^k) = 0$$

The method can be summarized by the algorithm described in figure 3.



**Figure 3** – The Hasofer-Lind-Rackwitz-Fiessler algorithm.

Once the reliability index obtained, the risk probability will be estimated by  $P_{risk} = \Phi(-\beta)$ . In this work, the gradient of  $H$  is expressed numerically by the centered difference scheme:

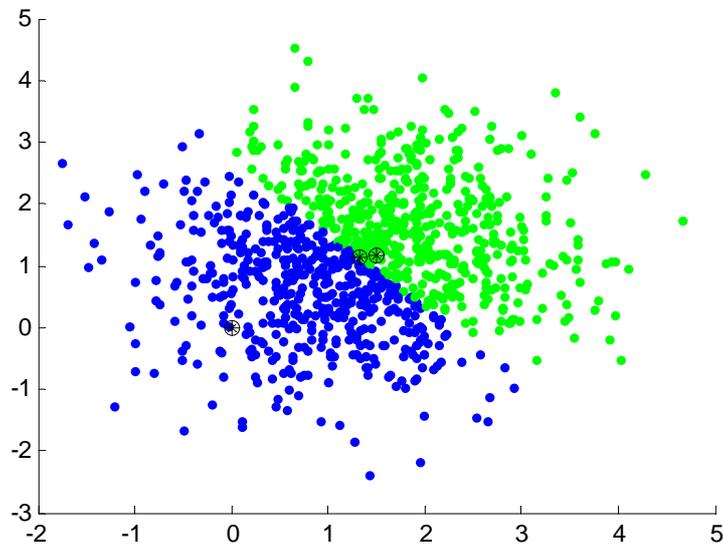
$$\frac{\partial H}{\partial u_i} \approx \frac{H(u_i + h) - H(u_i - h)}{2h}, \quad (10)$$

$h$  being small and  $2p$  additional evaluations of the limit state are required. A good choice of the step  $h$  is required to have a sufficiently precise gradient.

### 4.3 Application of Rollover Risk Evaluation for the HGV

We choose a scenario where the heavy goods vehicle is taking a bend. Linear variation of the steering angle is applied during the first seconds (from  $0^\circ$  to  $3^\circ$ ), afterwards it is maintained constant. The random variables taken into account are the speed  $v$  and the height of the centre of gravity  $h$  with means and standard deviations given by table 1. The limit state surface corresponds to the value  $LTR_{lim} = 1$ , for which we recall that a take-off one wheel is detected.

The HLRF algorithm is used to search for the design point  $P^*$  and compute the reliability index  $\beta$ . The accuracy is  $10^{-4}$ , the step 0.1 is used to numerically estimate the gradient. For a speed of 15m/s (54km/h), the algorithm converges after 5 iterations (25 calls of the limit state function). The reliability index found is 1.725, which corresponds to probability of wheel take-off of 4.14%. Figure 4 presents the evolution of the point  $P^*$ , and Monte Carlo simulations around this point.

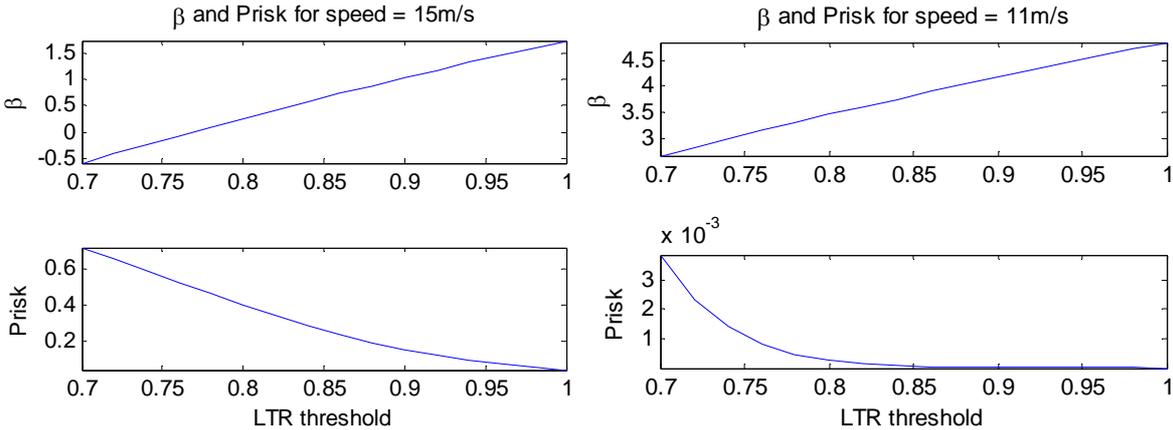


**Figure 4** – Results of searching  $P^*$  using HLRF algorithm and Monte Carlo simulations around  $P^*$  (dark for the safe domain, and bright for the unsafe domain).

According to figure 4 the limit state is almost linear. Then, first order method could be used to estimate the probability of rollover risk. The estimation by Monte Carlo simulations is 3.98% which is close to the result obtained by the HLRF algorithm.

Now, we evaluate rollover risk for small risk situations, which can be reached by increasing or decreasing the parameters of the heavy goods vehicle, or by increasing the standard

deviations of random variables. In our case, the vehicle speed is reduced to 11m/s (40km/s). For a threshold LTR=1 (wheel take-off risk), the algorithm converges after 7 iterations (35 call of the limit state function). The reliability index is about 4.85, which corresponds to a rollover risk probability of  $6.097 \cdot 10^{-7}$ . The probability is smaller than obtained with the previous scenario. The validation required an important number of Monte Carlo simulations of about  $10^9$  simulations, and leads to a rollover risk probability of  $5.84 \cdot 10^{-7}$ . The probability is close to that obtained by the developed algorithm where linear approximation of the limit state is supposed.



**Figure 5** – Reliability index and risk probability estimation for several thresholds of LTR, a) with a high rollover risk, b) with a small rollover risk.

The reliability indices and rollover risk probability according to LTR thresholds are shown in figure 5. The reliability index is seen to increase with LTR threshold. For small risk situation (figure 5b), reliability index variation is almost linear according to LTR thresholds, in opposition to the variation of the risk probability which have more nonlinearity. The reliability index gives then a good metric to the evaluation of rollover risk of the heavy goods vehicle in such a scenario.

**5. Conclusion**

In this paper, rollover risk of heavy goods vehicles is evaluated using a reliability approach. Because of the HGV dynamics and its interactions with the infrastructure, time intervenes in an explicit way in differential equations, which are stochastic in nature. To simplify the underlying problem, we opted for some simplifications. Random variables are dealt with instead of stochastic processes in order to solve deterministic models with random parameters. So, a static reliability method is used by choosing the maximum of the load transfer ratio as output for the limit state evaluation. Random variables are those corresponding to the most influential parameters on rollover risk, which are extracted after a sensitivity analysis. The risk is expressed in such a manner to get independent random variables. The initial results obtained encourage the use of the reliability-based approach to rollover risk evaluation of heavy goods vehicles.

Perspectives of this work concern reliability-based prevention in other scenarios with high rollover risk situations. The use of suitable laws of the random variables and a more

representative heavy vehicle model are required, which can be dealt with by an appropriate reliability method.

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