ROBUST CONTROL OF AN A-DOUBLE WITH ACTIVE DOLLY BASED ON STATIC OUTPUT FEEDBACK AND DYNAMIC FEED-FORWARD


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Abstract

A steering-based control is proposed for improving the lateral performance of an A-double combination with an active dolly. The controller is based on static output feedback combined with dynamic feed-forward and is designed to ensure an $\mathcal{H}_\infty$ performance objective in the face of parametric uncertainty. The synthesis is performed via linear matrix inequality (LMI) optimizations. Two feed-forward design methods are proposed and one of them is highlighted as the more rigorous approach for dealing with parametric uncertainty. The verification results confirm a significant reduction in rearward amplification of yaw rates and high speed transient off-tracking even when the dynamic feed-forward from the tractor steering angle accompanies the static feedback only from the articulation angles.

Keywords: Static output feedback , Dynamic feedforward, $\mathcal{H}_\infty$ synthesis , LMI based control, Rearward amplification, Active dolly

1. Introduction

Rapid growth in the transportation of goods has led to an increased demand for high capacity transport (HCT) vehicles, leading to raised concerns about environmental effects, road freight traffic and increased infrastructure usage. Increasing cost of fuel, congestions and gas emissions make long heavy vehicle combinations (LHVCs) as attractive alternatives to conventional heavy vehicle combinations (CHVCs) in goods transportation (H. Backman and R. Nordström (2002), J. Woodrooffe and L. Ash (2001)).

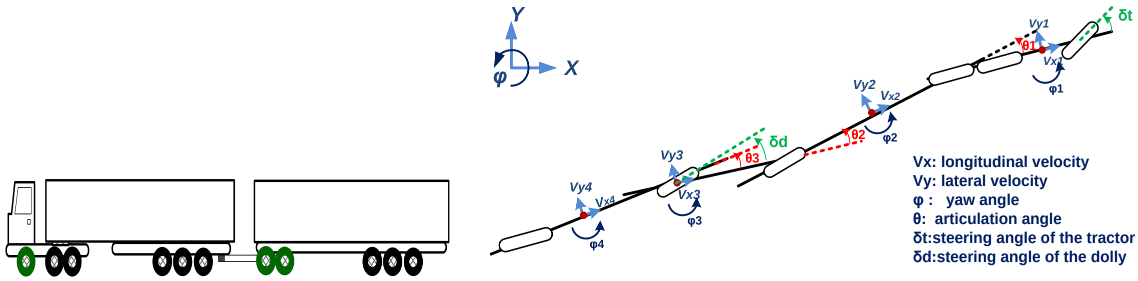
One major issue concerning LHVCs is their potential impacts on traffic safety. Road safety performance of LHVCs depends on the technical features such as power train and braking systems capability, lateral dynamical stability and maneuverability. Specific performance measures are introduced to assess the vehicle behaviour from various aspects. The most common measure is the so-called rearward amplification (RWA) of the yaw rate or lateral acceleration to assess the high speed lateral performance of HCT vehicles. Steering and braking based control are two main control strategies to improve the high-speed lateral performance of HCT vehicles. Several works have already proposed to reduce the yaw rate or lateral acceleration RWA for various HCT vehicles by using active trailer steering (see e.g. S. Kharrazi (2012) and the references therein).

The aim of this paper is to improve the high speed lateral performance of an A-double combination by controlling an active dolly. The dolly is a small unit in the A-double combination and therefore active dolly steering is more economical and convenient (M. M. Islam, L. Laine, and B. Jacobson (2015)). The potential improvements by controlling the dolly have been studied in (M. S. Kati, L. Laine, and B. Jacobson (2014)) by using a simple feedback control strategy. M. M. Islam, L. Laine, and B. Jacobson (2015) have proposed a feed-forward controller based on a nonlinear inversion technique and investigated controlled system sensitivity against the uncertainty in the steering actuator parameters.

In this paper, we propose a combined static output feedback and dynamic feed-forward controller which is designed to be robust against the variations in the steering actuator parameters. Such a scheme is shown to improve the lateral performance of an A-double combination as justified by frequency domain analysis and time-domain simulations.

2. Vehicle Model

An A-double combination consists of a tractor as the lead unit and two semitrailers linked together by a dolly converter unit as shown in Figure 1. Since the dolly is the smallest among all A-double units, its axles are chosen to be steerable for yaw rate control of the A-double, while the axles of the other units stay unchanged. The A-double is considered with a total weight of 80 tons and a total length of about 31.5 meters. Figure 1 shows an A-double with steerable axles identified by green tyres.



(a) A-double (b) Bicycle model

Figure 1 – A-double combination schematic and bicycle model.

The controller is designed based on a linear vehicle model without accounting for the roll dynamics and load transfer. The linear vehicle model is considered to be accurate under the assumption that steering and articulation angles are small. The schematic diagram of the linear vehicle model is depicted in Figure 1, where the axle groups in the semitrailers and the dolly are lumped together into a single axle in the middle of each axle group. We assume that all parameters of the vehicle are known and fixed except the dolly steering actuator parameters. A linearized single-track model of the A-double is derived by using lagrangian method and expressed as

$$\mathcal{M}\ddot{q}(t) + \mathcal{C}\dot{q}(t) + \mathcal{K}q(t) = \mathcal{F}_{\delta_d}\delta_d(t) + \mathcal{F}_{\delta_t}\delta_t(t), \tag{1}$$

where $\mathcal{M}$ is the mass matrix, $\mathcal{C}$ is the damping matrix and $\mathcal{K}$, $\mathcal{F}_{\delta_d}$ and $\mathcal{F}_{\delta_t}$ are the stiffness matrices. δ_d is the dolly steering angle that will be obtained as the output of the actuator, while δ_t is the tractor steering angle viewed as the disturbance input. The generalized coordinate vector is identified by $q^T = [y_1\, \varphi_1\, \theta_1\, \theta_2\, \theta_3]$ in reference to Figure 1. Now considering the state vector as $x_q^T \triangleq [q^T \quad \dot{q}^T]$, a state-space model of the system in (1) can be written in the form of

$$\underbrace{\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix}}_{\dot{x}_q} = \underbrace{\begin{bmatrix} 0 & I \\ -\mathcal{M}^{-1}\mathcal{K} & -\mathcal{M}^{-1}\mathcal{C} \end{bmatrix}}_{\mathscr{A}} \underbrace{\begin{bmatrix} q \\ \dot{q} \end{bmatrix}}_{x_q} + \underbrace{\begin{bmatrix} 0 \\ \mathcal{M}^{-1}\mathcal{F}_{\delta_d} \end{bmatrix}}_{\mathscr{B}} \delta_d + \underbrace{\begin{bmatrix} 0 \\ \mathcal{M}^{-1}\mathcal{F}_{\delta_t} \end{bmatrix}}_{\mathscr{H}} \delta_t. \tag{2}$$

Two states y_1 and φ_1 are removed from x_q to obtain the state-space model to be used in the design (M. Levén, A. Sjöblom, M. Lidberg and B. Schofield (2011)). We stress that this is possible thanks to the structure of the matrix $\mathcal{K}$. As a result, the state vector is formed as

$$x_p = [\theta_1\ \theta_2\ \theta_3\ V_{y1}\ \dot{\varphi}_1\ \dot{\theta}_1\ \dot{\theta}_2\ \dot{\theta}_3]^T. \tag{3}$$

By removing the relevant row blocks from all matrices and also the relevant column block from $\mathcal{A}$, we obtain a state-space model of the plant that will be considered in the design as follows:

$$\dot{x}_p = A_p x_p + B_p \delta_d + H_p \delta_t. \tag{4}$$

The dolly steering actuator model consists of two parts: a first order filter and a time delay as shown in Figure 2.b. The first order filter is characterized by a time constant of τ_a, while the second part is modeling a transport delay of τ_d. The transfer function of the time delay is approximated by a first order Padé-approximation as follows:

$$e^{-s\tau_d} \approx \frac{1 - 0.5\tau_d}{1 + 0.5\tau_d}. \tag{5}$$

We then introduce a state space realization of the actuator model as

$$\dot{x}_a = \underbrace{\begin{bmatrix} -2/\tau_d & 0 \\ 1/\tau_a & -1/\tau_a \end{bmatrix}}_{A_a(\sigma)} x_a + \underbrace{\begin{bmatrix} 4/\tau_d \\ -1/\tau_a \end{bmatrix}}_{B_a(\sigma)} u, \quad \delta_d = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C_a} x_a + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D_a} u, \tag{6}$$

where u is the control input to be designed. Note that the actuator dynamics depend rationally on τ_a and τ_d, which will be considered as uncertain. In order to simplify the type of dependence, we introduce two new parameters as $\sigma_1 = 1/\tau_a$ and $\sigma_2 = 1/\tau_d$. In this fashion, we obtain an equivalent representation of the system with affine dependence on $\sigma = (\sigma_1, \sigma_2)$ as

$$\dot{x}_a = A_a(\sigma)x_a + B_a(\sigma)u, \quad \delta_d = C_a x_a + D_a u. \tag{7}$$

It is assumed that the uncertain parameter vector is known to be in the following set:

$$\mathbf{\Delta} \triangleq \{\sigma = (\sigma_1, \sigma_2) : \sigma_i \in [\sigma_i^{\min}, \sigma_i^{\max}], i = 1,2\}. \tag{8}$$

In addition, to characterize typical driver behaviour, we assume that the frequency content of δ_t is concentrated in a specific frequency range. Therefore, we introduce an artificial bandpass filter with a realization as

$$\dot{x}_b = A_b x_b + B_b w, \quad \delta_t = C_b x_b + D_b w, \tag{9}$$

where x_b and w are the artificial state vector and disturbance input respectively. In this paper, we consider a simple second-order band-pass filter. With the Laplace transform of $w(t)$ represented as $\hat{w}(s)$, we express the tractor steering angle as

$$\hat{\delta}_t(s) = \underbrace{\frac{2\zeta\omega_c s}{s^2 + 2\zeta\omega_c s + \omega_c^2}}_{\mathcal{W}(s)} \hat{w}(s), \tag{10}$$

where ω_c is identified as the frequency at which the filter gain is one. The 3-dB bandwidth of this filter is identified as $[\omega_l, \omega_h]$, where the lower and upper limits are given by

$$\omega_l = \left(1 + 2\zeta^2 - \sqrt{(1+2\zeta^2)^2 - 1}\right)\omega_c, \quad \omega_h = \left(1 + 2\zeta^2 + \sqrt{(1+2\zeta^2)^2 - 1}\right)\omega_c. \tag{11}$$

The bandwidth of the filter needs to be chosen based on typical and realistic driver inputs. We now append the steering actuator dynamics and the bandpass filter to the dynamics of the plant to obtain a complete state-space description as

$$
\begin{aligned}
\dot{x} &= \underbrace{\begin{bmatrix} A_p & B_pC_a & H_pC_b \\ 0 & A_a(\sigma) & 0 \\ 0 & 0 & A_b \end{bmatrix}}_{A(\sigma)} \underbrace{\begin{bmatrix} x_p \\ x_a \\ x_b \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} H_pD_b \\ 0 \\ B_b \end{bmatrix}}_{H} w + \underbrace{\begin{bmatrix} B_pD_a \\ B_a(\sigma) \\ 0 \end{bmatrix}}_{B(\sigma)} u, \\
z &= \underbrace{\begin{bmatrix} C_p & D_pC_a & G_pC_b \end{bmatrix}}_{C} x + \underbrace{G_pD_b}_{G} w + \underbrace{\begin{bmatrix} D_pD_a \end{bmatrix}}_{D} u, \\
y &= \underbrace{\begin{bmatrix} S_p & 0 & R_pC_b \end{bmatrix}}_{S} x + \underbrace{\begin{bmatrix} R_pD_b \end{bmatrix}}_{R} w,
\end{aligned}
\tag{12}
$$

where $z = C_px_p + D_p\delta_d + G_p\delta_t$ is the signal used for performance evaluation and $y = S_px_p + R_p\delta_t$ is the measured output. We emphasize that the parameter dependence is obtained only in the A and B matrix in this model.

As we will detail in the next section, the design objective will be to synthesize a fixed controller such that the controlled system behaves in a desirable way for all values of $\sigma = (\sigma_1, \sigma_2)$ within the domain given in (8).

3. Control Design

The objective of the lateral controller is to suppress undesired yaw rate rearward amplification (RWA) in the towed units by active steering of the dolly in a single lane change maneuver. We propose a static output feedback controller plus a dynamic feed-forward for this purpose. The closed loop system schematic diagram is shown in Figure 2.a. In this section, we first provide a suitable formulation of the problem and then present possible synthesis procedures based on LMI optimization.

The objective of the controller is to determine the required steering angle for the dolly to suppress the yaw rate RWA of the last semitrailer. Requiring this might cause an increase in the yaw rate of the dolly. In order to avoid high RWA both in the dolly and the last semitrailer, we choose the performance output as

$$
z = \begin{bmatrix} \lambda r_3 \\ (1-\lambda) r_4 \end{bmatrix}. \tag{13}
$$

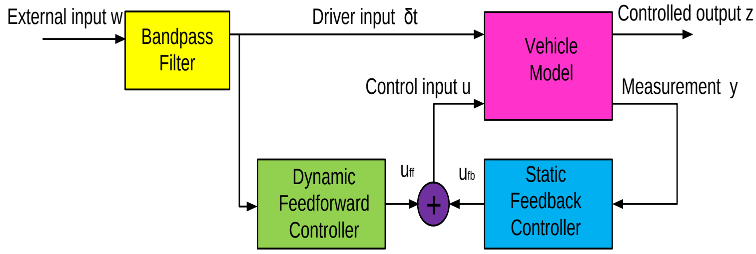
where $\lambda \in [0,1)$ is a scalar that can be used to adjust the relative emphasis on the dolly and the second semitrailer.

The control law for the synthesis problem is given by

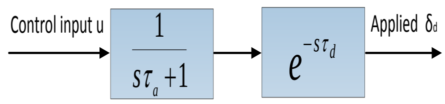
$$
u(t) = u_{fb}(t) + u_{ff}(t), \tag{14}
$$

where u_{fb} is the static output feedback control input and u_{ff} is the dynamic feed-forward control input. We consider generating the feedback control input u_{fb} as

$$
u_{fb}(t) = K_{fb}y(t), \tag{15}
$$



(a) General system



(b) Dolly steering actuator model

Figure 2 – Schematic diagrams

where K_{fb} is the feedback gain vector to be designed. On the other hand, u_{ff} is obtained by passing δ_t through a dynamic filter described by

$$\dot{x}_f = A_f x_f + B_f \delta_t, \qquad u_{ff} = C_f x_f + D_f \delta_t, \tag{16}$$

where x_f represents the filter state and (A_f, B_f, C_f, D_f) represents a realization of the filter to be found. We can hence express the feed-forward control input as

$$\hat{u}_{ff}(s) = \underbrace{\left(C_f(sI - A_f)^{-1}B_f + D_f\right)}_{\mathcal{K}_{ff}(s)} \hat{\delta}_t(s). \tag{17}$$

Our design procedure is composed of two steps. In the first step, the static output feedback gain K_{fb} is obtained. This is then used to find the closed-loop system description in the absence of feed-forward control. This description is used in the second step in which $\mathcal{K}_{ff}(s)$ is designed.

3.1 First Step: Robust Static Output Feedback Design

We consider a static output synthesis with an $\mathcal{H}_\infty$ performance objective. The $\mathcal{H}_\infty$ synthesis aims at ensuring bounds on the energy gain from the disturbance to the performance output. An $\mathcal{H}_\infty$ synthesis problem can be formulated for our system as follows: Given the state-space description in (12), find a gain vector K_{fb} such that, with the control input generated as in (14), the closed-loop system is stable and satisfies the following condition for all $\sigma \in \mathbf{\Delta}$:

$$\|z\|_2 < \gamma \|w\|_2, \forall w \text{ with } 0 < \|w\|_2 < \infty \text{ and } x(0) = 0. \tag{18}$$

At this point, we recall the definition of the $\mathcal{L}_2$-norm as $\|w\|_2 \triangleq \sqrt{\int_0^\infty w(t)^T w(t) dt}$. In a typical synthesis, the goal would be to minimize γ.

Let us now describe a procedure for $\mathcal{H}_\infty$ synthesis as adapted from (H. Köroğlu and P. Falcone (2014)). We also introduce the set of extreme points of the parameter domain $\mathbf{\Delta}$ based on the minimum and maximum values of σ_i's as

$$\mathbf{\Pi} \triangleq \{\sigma^i : i = 1, \ldots, \eta\}. \tag{19}$$

In our case with two uncertain parameters, we will have $\eta = 4$ extreme points as

$$(\sigma_1^{\min}, \sigma_2^{\min}), (\sigma_1^{\min}, \sigma_2^{\max}), (\sigma_1^{\max}, \sigma_2^{\min}), (\sigma_1^{\max}, \sigma_2^{\max}). \tag{20}$$

We also introduce the associated values of the A matrix as follows:

$$A^i \triangleq A(\sigma^i) \quad and \quad B^i \triangleq B(\sigma^i). \tag{21}$$

The synthesis can then be realized via the following steps (see H. Köroğlu and P. Falcone (2014)):

1) Consider a grid for a positive scalar ϕ with $\phi_1 = \phi_{\min} > 0$ and $\phi_\mu = \phi_{\max} > \phi_{\min}$.
2) For each $\phi_j, j = 1,\ldots,\mu$, find γ_j by solving

$$\gamma_j = \arg\min\{\gamma : \mathcal{N}^i(\phi_j) \prec 0, i = 1,\ldots,\eta; Y \succ 0\} \tag{22}$$

where $\mathcal{N}^i$ is defined in terms of ϕ and the matrix variables $Y = Y^T \in \mathbb{R}^{k\times k}$, $W \in \mathbb{R}^{m\times m}$, $N \in \mathbb{R}^{1\times m}$ as

$$\mathcal{N}^i(\phi) \triangleq \mathsf{He}\begin{bmatrix} -\phi W & \phi(SY-WS) & \phi R & 0 \\ BN & A^iY+B^iNS & H & 0 \\ 0 & 0 & -\frac{\gamma}{2}I & 0 \\ DN & CY+DNS & G & -\frac{\gamma}{2}I \end{bmatrix}. \tag{23}$$

Here we used $\mathsf{He}\mathcal{M} \triangleq \mathcal{M} + \mathcal{M}^T$ to simplify the expression of the LMI condition. The state and output dimensions are represented as k and m respectively.
3) Find the minimum γ as $\gamma_{\min} = \min\{\gamma_j : j = 1,\ldots,\mu\}$ and construct K_{fb} by using (W,N) obtained from the optimization problem associated with $\gamma_{\min}$ as follows:

$$K_{fb} = NW^{-1}. \tag{24}$$

By applying the feedback control obtained from this step, a new state space realization can be obtained for the controlled plant combined with the actuator as follows:

$$\begin{aligned} \dot{x}_c = {} & \underbrace{\begin{bmatrix} A_p + B_pD_aK_{fb}S_p & B_pC_a \\ B_aK_{fb}S_p & A_a \end{bmatrix}}_{A_c(\sigma)} \underbrace{\begin{bmatrix} x_p \\ x_a \end{bmatrix}}_{x_c} + \underbrace{\begin{bmatrix} H_pC_b + B_pD_aK_{fb}R_pC_b \\ B_aK_{fb}R_pC_b \end{bmatrix}}_{H_b(\sigma)} x_b \\ & + \underbrace{\begin{bmatrix} H_pD_b + B_pD_aK_{fb}R_pD_b \\ B_aK_{fb}R_pD_b \end{bmatrix}}_{H_c(\sigma)} w + \underbrace{\begin{bmatrix} B_pD_a \\ B_a \end{bmatrix}}_{B_c(\sigma)} u_{ff}, \end{aligned} \tag{25}$$

$$\begin{aligned} z = {} & \underbrace{\begin{bmatrix} C_p + D_pD_aK_{fb}S_p & D_pC_a \end{bmatrix}}_{C_c} x_c + \underbrace{\begin{bmatrix} G_pC_b + D_pD_aK_{fb}R_pC_b \end{bmatrix}}_{G_b} x_b \\ & + \underbrace{\begin{bmatrix} G_pD_b + D_pD_aK_{fb}R_pD_b \end{bmatrix}}_{G_c} w + \underbrace{D}_{D_c} u_{ff}, \end{aligned} \tag{26}$$

In this description, u_{ff} is the feed-forward control input which is left to be designed in the next step.

3.2 Second Step: Dynamic Feed-Forward Design

We propose two alternative methods for dynamic feed-forward design. The first method is applicable to a nominal model, while the second one can be used for an uncertain system as well.

3.2.1 Method 1

In this method, we reformulate the problem as an $\mathcal{H}_\infty$ model matching problem and design the dynamic feedforward via standard dynamic output feedback $\mathcal{H}_\infty$ synthesis (C. Scherer, P. Gahinet, and M. Chilali (1997)). For this purpose, we fix σ to its nominal (typical or average) value and consider the design for a single model. The problem is then formulated as a standard $\mathcal{H}_\infty$ model matching problem. To this end, we first find the expression of z in the Laplace domain as

$$\hat{z}(s) = \underbrace{(C_{cl}(sI - A_{cl})^{-1}H_{cl} + G_c)}_{\mathcal{G}(s)}\hat{w}(s) + \underbrace{(C_{cl}(sI - A_{cl})^{-1}B_{cl} + D_c)}_{\mathcal{T}(s)}\hat{u}_{ff}(s).$$

where we have $A_{cl} = \begin{bmatrix} A_c & H_b \\ 0 & Ab \end{bmatrix}$, $H_{cl} = \begin{bmatrix} H_c \\ B_b \end{bmatrix}$, $B_{cl} = \begin{bmatrix} B_c \\ 0 \end{bmatrix}$ and $C_{cl} = \begin{bmatrix} C_c & G_b \end{bmatrix}$. By now recalling the expression of $\hat{u}_{ff}$ as in (17), we end up with the following expression:

$$\hat{z}(s) = \left(\mathcal{G}(s) + \mathcal{T}(s)\mathcal{K}_{ff}(s)\mathcal{W}(s)\right)\hat{w}(s). \tag{27}$$

The standard $\mathcal{H}_\infty$ model-matching problem aims at finding a stable transfer function $\mathcal{K}_{ff}$ by performing the following $\mathcal{H}_\infty$-norm minimization:

$$\gamma_{opt} := \min_{\mathcal{K}_{ff}(s)\ \text{stable}} \left\|\mathcal{G}(s) + \mathcal{T}(s)\mathcal{K}_{ff}(s)\mathcal{W}(s)\right\|_\infty. \tag{28}$$

The transfer matrices $\mathcal{G}$ and $\mathcal{T}$ are required to be stable and proper, which will be the case in our problem. This problem is easily reformulated as an $\mathcal{H}_\infty$ synthesis based on dynamic output feedback, for which a standard Matlab function can be used.

3.2.2 Method 2

In the second method, we use the approach proposed in (H. Köroğlu and P. Falcone (2014)) by a modification which is including a weighting filter for the external disturbance. The feed-forwarded signal is basically a filtered version of external disturbance in the system. In order to reformulate the problem as in (H. Köroğlu and P. Falcone (2014)), we first combine the dynamics of the band-pass filter with the dynamics of the feed-forward filter as follows:

$$\dot{\hat{x}}_f = \underbrace{\begin{bmatrix} A_b & 0 \\ B_f C_b & A_f \end{bmatrix}}_{\hat{A}_f}\underbrace{\begin{bmatrix} x_b \\ x_f \end{bmatrix}}_{\hat{x}_f} + \underbrace{\begin{bmatrix} B_b \\ B_f D_b \end{bmatrix}}_{\hat{B}_f} w, \tag{29}$$

$$u_{ff} = \underbrace{\begin{bmatrix} D_f C_b & C_f \end{bmatrix}}_{\hat{C}_f}\hat{x}_f + \underbrace{\begin{bmatrix} D_f D_b \end{bmatrix}}_{\hat{D}_f} w. \tag{30}$$

We next re-express the dynamics of the controlled plant from (25)-(26) as

$$\dot{x}_c = A_c x_c + \underbrace{\begin{bmatrix} H_b & 0 \end{bmatrix}}_{\hat{H}_b}\hat{x}_f + H_c w + B_c u_{ff}, \tag{31}$$

$$z = C_c x_c + \underbrace{\begin{bmatrix} G_b & 0 \end{bmatrix}}_{\hat{G}_b}\hat{x}_f + G_c w + D_c u_{ff}. \tag{32}$$

and thereby introduce the extended matrices $\hat{H}_b$ and $\hat{G}_b$. By then applying the same approach as in (H. Köroğlu and P. Falcone (2014)), we obtain an LMI condition for the performance objective in (18) as follows:

$$\mathrm{He}\begin{bmatrix} A_cY & \hat{H}_b + A_cV - V\hat{A}_f & \hat{H}_c + B_c\hat{D}_f - V\hat{B}_f & 0 \\ 0 & X\hat{A}_f & X\hat{B}_f & 0 \\ 0 & 0 & -\frac{\gamma}{2}I & 0 \\ C_cY & \hat{G}_b + G_cV + D_c\hat{C}_f & G_c + D_c\hat{D}_f & -\frac{\gamma}{2}I \end{bmatrix} \prec 0. \tag{33}$$

In this condition $Y \succ 0$ and $X \succ 0$ are symmetric positive-definite matrix variables, while V is an arbitrary matrix variable. The dimensions of these matrices can be identified from compatibility. On the other hand, $\hat{C}_f$ and $\hat{D}_f$ are structured matrix variables that depend on C_f and D_f as identified from (30). We consider that (A_f,B_f) and thereby $(\hat{A}_f,\hat{B}_f)$ are fixed while C_f and D_f are to be designed. We use the same form of the realization given in (H. Köroğlu and P. Falcone (2014)) where the transfer function $\mathcal{K}_{ff}(s)$ is expressed as

$$\mathcal{K}_{ff}(s)=C_f(sI-A_f)B_f+D_f=D_f+\sum_{i=1}^{l}C_f{}^i(s+\psi)^{-i}. \tag{34}$$

The variables ψ and l are the pole and the order of the filter to be fixed by the designer beforehand. Note that the LMI in (33) has affine dependence on the uncertain parameter σ via the system matrices in (31)-(32), which is suppressed for simplicity. Hence the parameter-dependent LMI is rendered finite-dimensional simply be imposing it at the extreme points.

4. Simulation Results

In this section, the synthesis procedures developed in the previous sections are applied to a linear vehicle model with vehicle parameters from (M. M. Islam, L. Laine, and B. Jacobson (2015)) and associated simulation results are provided. The A-double is equipped with an active dolly where both axles are steerable with the same amount of steering angle. A single lane change manoeuvre is performed at a longitudinal velocity of 80 Km/h. A sinusoidal steering input is chosen for the input w with an amplitude of 3 deg and a frequency of 0.4 Hz. It is assumed that only the articulation angles are available, i.e. $y = \begin{bmatrix}\theta_1 & \theta_2 & \theta_3\end{bmatrix}^T$.

Recall that the objective of the controller is to calculate the required $u(t)$ in (14) to minimize the yaw rate of the dolly and the second semitrailer. Thus, the control design is provided for the performance indicator in (13) with the weighting coefficient $\lambda = 0.5$. With this choice, we place the same emphasis on the yaw rate of both the dolly and the second semitrailer.

The bandpass filter in (10) is used to characterize the tractor steering input with a fixed bandwidth of $[\omega_l,\omega_h]=[2\pi\times 0.2, 2\pi\times 0.6]$ for all cases. The values $\tau_a = 0.35$ and $\tau_d = 0.1$ are chosen for the nominal system from the work in (M. M. Islam, L. Laine, and B. Jacobson (2015)). For the uncertain steering actuator model, $\tau_a \in [0.3,0.4]$ and

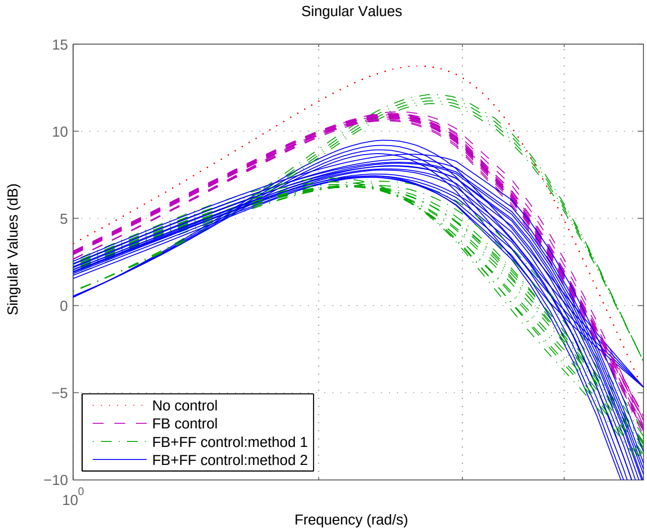


Figure 3 – Singular value plots for robust static output feedback design and two dynamic feedforward design methods

$\tau_d \in [0.05, 0.5]$ are chosen.

In order to synthesize the output feedback controller in the first step, we first find the minimum value of γ via an interval search over ϕ for both nominal and uncertain system. The minimum γ value and the associated gain vector are computed as follows:

$$\gamma = 3.763\ ,\quad K_{fb} = \begin{bmatrix} -0.487 & -0.499 & -0.083 \end{bmatrix}.$$

In the second step, the dynamic feed-forward filter $\mathcal{K}_{fb}(s)$ is computed with two alternative methods as follows:

$$\begin{aligned}
\mathcal{K}^1_{ff}(s) =& \frac{357.94(s+20.1)(s+8.7)(s+4.75)(s+1.67(s+0.02}{(s+20)(s+0.001)(s^2+3.52s+4.74)(s^2+3.06s+6.94)} \times \\
& \frac{(s^2+2.18s+4.3)(s^2-3.81s+24.91)(s^2+10.59s+79.95)}{(s^2+2.23s+7.33)(s^2+8.993s+34.68)(s^2+208.4s+21730)} \\
\mathcal{K}^2_{ff}(s) =& \frac{1.8389(s^2-3.13s+27.63)}{(s+5)^2}.
\end{aligned}$$

Figure 3 illustrates the singular value plots of the transfer function from δ_t to z in all the cases. These are obtained as overlay plots for different values of the parameters τ_a and τ_d within the considered ranges. We observe from this figure that the static output feedback controller already leads to some reduction in the maximum singular value. Nevertheless, by adding dynamic feedforward, even further reduction is obtained. Although the filter obtained with the first method leads to significant reduction in the nominal case, the maximum singular value increases for some parameters within the considered range. On the other hand, we observe a robust behavior with the second filter in that the maximum singular value is kept desirably small in all the cases.

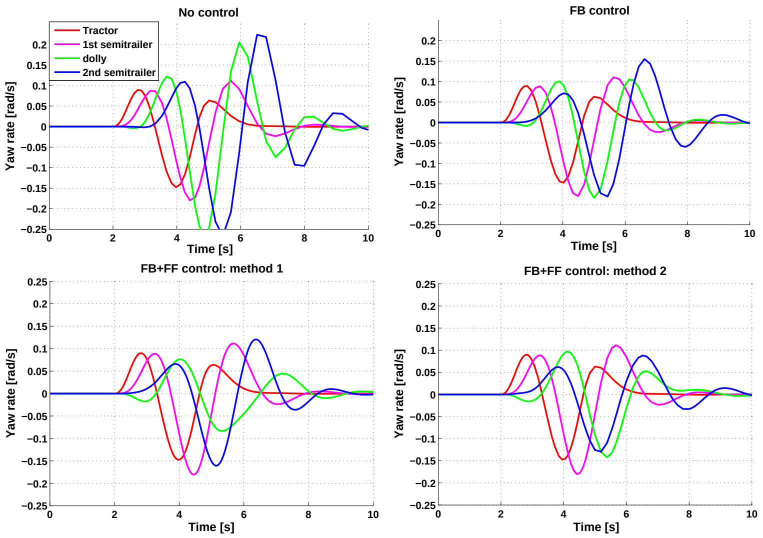


Figure 4 – Yaw rates of A-double units through single lane change

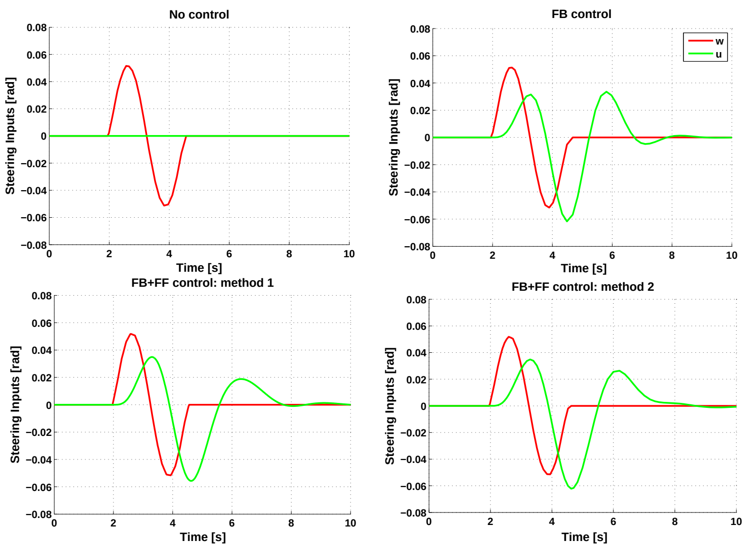


Figure 5 – Steering inputs for passive and active dolly steering cases

The controllers are also tested in time domain simulations, whose results are presented in Figures 4 and 5. For ease of evaluation, we also provide in Table 1 the yaw rate RWA of the dolly and the last semitrailer in four different cases: passive (i.e. uncontrolled) dolly steering case and three different active (i.e. controlled) dolly steering cases. We observe that the performance is improved significantly with combined static feedback and dynamic feed-forward if compared to the case in which feed-forward is not applied.

Table 1 – Yaw rate RWA of the dolly and the last semitrailer for passive and active dolly steering cases for the nominal system

Synthesis methods	Lat. acc. RWA3	Lat. acc. RWA4
Without Controller	1.79	1.81
FB controller	1.25	1.23
FB+FF controller, method 1	0.57	1.1
FB+FF controller, method 2	1.11	0.75

5. Conclusions

In this paper, a static output feedback controller combined with dynamic feed-forward is proposed for the control of an A-double combination. The designs are based on the $\mathcal{H}_\infty$ synthesis approach. Both feedback and feed-forward designs are adapted from previous works with some suitable modifications. Two alternative feed-forward design approaches are proposed and compared. The second method is applicable to uncertain systems and provides filters with fixed order and poles. As illustrated by our design example, one can obtain robust performance by applying the second method with significantly reduced orders.

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